The COMPLETE MEASURER: OR, THE *Whole Art of Measuring.*

In Two Parts.

The First PART teaching DECIMAL ARITHMETIC, with the Extraction of the *Square* and *Cube-Roots*.

Also the *Multiplication* of *Feet* and *Inches*, commonly call'd *Cross-Multiplication*.

The Second PART teaching to Measure all Sorts of *Superficies* and *Solids*, by *Decimals*, by *Cross-Multiplication*, and by *Scale* and *Compasses*. Also the Works of several *Artificers* relating to *Building*; and the *Measuring* of *Board* and *Timber*: Shewing the common Errors. And some *Practical Questions*.

The FIFTH *EDITION; to which is added, an APPENDIX,* 1. *Of* Gaging, 2. *Of* Land-Measuring.

Very Useful for all Tradesmen, especially Carpenters, Bricklayers, Plaisterers, Painters, Joyners, Glasiers, Masons, *&c.*

By WILLIAM HAWNEY, *Philomath.*

Recommended by the Rev. Dr. John Harris, F. R. S.

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15
7
105
21

THE PREFACE.

HAVING perus'd ſeveral Books concerning the Menſuration of Superficies and Solids, and the Works of Artificers relating to Building; but not finding any one Book ſo perfect, as to give any tolerable Satiſfaction to a Learner; and I having prac-

tis'd and taught Meaſuring for ſeveral Years, and thereby gain'd Experience and Knowledge in that Art, having learned ſome Things from one Author, and ſome Things from another, I began to think of digeſting my Thoughts into ſome ſuch Method as might give a Learner full Satisfaction, without being at the Charge of buying ſo many Books; and being importun'd thereunto by ſome Friends, I fell to work, and at laſt brought them to that Perfection you here find the following WORK.

1. As to the DECIMAL ARITHMETIC, I have been as brief as the Matter would well bear, to make it plain.

2. As to the Multiplying of Feet and Inches, commonly call'd CROSS-MULTIPLICATION; my Method differs from that which is uſually taught in other Authors, as being (I think) much ſhorter and plainer.

3. IN

3. In meaſuring of Superficies and Solids, I have given the Demonſtration of the Rules, which I thought might be very acceptable to the Ingenious; for indeed I always look upon the Writing of a Rule without a Demonſtration, (in any Part of the Mathematicks) to be but lame and defective; and for want of knowing the Reaſon of the Rule, a Learner may commit great Errors; beſides, when a Learner knows the Reaſon of the Rules, he may retain them better in his Memory. The Rule for meaſuring a Priſmoid and Cylindroid, I had out of Mr. *Everard's Art of Gaging*; but the Reaſon he does not ſhew, neither have I found it in any other Author; but that the Method is true, I have endeavour'd to make plain.

The Demonſtrations of the Rules for finding an Area of an Ellipſis and Parabola; alſo the Demonſtration of the Rules for finding the ſolid Content of the Fruſtum of a Cone and Pyramid, the Solidity of a Globe, of a Spheroid, a Parabolick Conoid, and of a Parabolick Spindle, and their Fruſtums, I had from the ingenious Mr. *Ward's Young Mathematici-*

an's

an's Guide; where the curious and ingenious Reader may ſee many other Demonſtrations algebraically perform'd: I have alſo demonſtrated the Rule for finding the Solidity of a Globe out of *Pardie's Elements of Geometry*, (Book the Vth, Art. the 33d) publiſh'd in *Engliſh*, with many Additions, by the Reverend Dr. *Harris*, *F. R. S.* and the ſame is alſo done out of *Sturmius*'s *Matheſis Enucleata*; ſo that the ingenious Reader may uſe which of thoſe Ways he likes beſt.

THE Scale ſuppos'd to be uſed in all the Operations, is the Line of Numbers, commonly call'd, *Gunter*'s *Line*, which is upon the ordinary Two Feet, or Eighteen Inch-Rules, commonly uſed by the Carpenters, Maſons, *&c.* becauſe I thought it needleſs, as well as impertinent, to write the Uſe of Sliding-Rules, or any other particular Scales, they being ſufficiently treated of by ſeveral Authors, *viz.* by the above-nam'd Mr. *Everard*, in his *Art of Gaging* above-mention'd, where you have the Uſe of a Sliding-Rule in Arithmetic, Geometry, in meaſuring of Superficies and Solids, Gaging, *&c.* Likewiſe Mr. *Hunt* has wrote largely of the

Uſe

Uſe of his Sliding-Rule, in Arithmetic, Geometry, Trigonometry, Gaging, Dyalling, &c. There are ſeveral others who have explain'd the Uſe of their own Rules; ſo that the more curious Readers may find full Satisfaction in thoſe Authors.

One Thing I have omitted in the Book, which I think may not be very improperly inſerted in this Place; that is, how to find a Number upon the Line. If the Number you would find, conſiſts only of Units, then the Figures upon the Line repreſent the Number ſought: Thus, if the Number be 1, 2, 3, &c. then 1, 2, 3, &c. upon the Line, repreſent the Number ſought. But if the Number conſiſts of two Figures, that is, of Units and Tens, then the Figure upon the Rule ſtands for Tens, and the larger Diviſions ſtand for Units; thus, if 34 were to be found upon the Line, the Figure 3 upon the Line is 30, and 4 of the large Diviſions (counted forward) is the Point repreſenting 34, and if 340 were to be found, it will be at the ſame Point upon the Line; and if 304 were to

be ſound, then the 3 upon the Line is 300, and 4 of the ſmaller Diviſions (counted forward) is the Point repreſenting 304. If the Number conſiſts of four Places, or Thouſands, then the Figure upon the Line ſtands for Thouſands, and the larger Diviſions are Hundreds, the leſſer Diviſions are Tens, and the tenth Parts of thoſe leſſer Diviſions are Units. Thus, if 2735 were to be found, then the 2 is 2000; and the 7 larger Diviſions (counted forward) is 700 more; and 3 of the leſſer Diviſions is 30 more; and half of one of the leſſer Diviſions is 5 more, which is the Point repreſenting 2735. You muſt remember, that between each Figure upon the Line there are 10 Parts, which I call the larger Diviſions; and each of thoſe larger Diviſions are ſubdivided (or ſuppoſed ſo to be) into 10 other Parts, which I call the ſmaller Diviſions; and each of thoſe Parts ſuppoſed to be ſubdivided again into 10 other Parts, *&c.* You muſt alſo remember, that if 1, in the Middle of the Line, ſtands only for 1, then 1 at the upper End will be 10, and 1 at the lower End will only be $\frac{1}{10}$; but if 1 at the lower End

ſigni-

ſignifies 1, then 1 in the Middle ſtands for 10, and 1 at the upper End is 100, *&c.*

THERE is one Thing more which I would have my Reader to underſtand, and that is, How to find all ſuch proportional Numbers made Uſe of in the Proportions about a Circle, and of a Cylinder, and in other Places; which Thing may be of good Uſe, to know how to correct a Number which may happen to be falſe printed, or to enlarge any Number to more decimal Places, for more Exactneſs; for though I have mention'd what ſuch Numbers are, yet I have not ſhewn how to find them, which a Learner may be a little at a Nonplus to do; though they are eaſily found by the Rules there laid down. I ſhall therefore give two or three Examples, in this Place, of finding ſuch Numbers, which may enable my Reader to find out the reſt.

AND, firſt, let it be requir'd to find the Area of a Circle, whoſe Diameter is an Unit.

By the Proportion of *Van Culen*, if the Diameter be 1, the Circumference will be 3.1415926, &c. whereof 3.1416 is sufficient in most Cases. Then the Rule teaches to multiply half the Circumference by half the Diameter, and the Product is the Area: That is, Multiply 1.5708 by .5, (*viz.* half 3.1416 by half 1) and the Product is .7854, which is the Area of the Circle, whose Diameter is 1.

Again, if the Area be requir'd, when the Circumference is 1, first, find what the Diameter will be, thus, as 3.1416 : to 1, : : so is 1 to .318309, which is the Diameter when the Circumference is 1. Then multiply half .318309 by half 1; that is, .159154 by .5, and the Product is .079577, which is the Area of a Circle whose Circumference is 1.

If the Area be given, to find the Side of the Square equal, you need but extract the Square Root of the Area given, and it is done: So the Square Root of .7854 is .8862, which is the Side of a

Square equal when the Diameter is 1. And if you extract the Square Root of .079577, it will be .2821, which is the Side of the Square equal to the Circle, whose Circumference is 1.

If the Side of a Square within a Circle, be requir'd, if you square the Semidiameter, and double that Square, and out of that Sum extract the Square Root, that shall be the Side of the Square which may be inscrib'd in that Circle; so, if the Diameter of the Circle be 1, then the Half is .5; which squar'd, is .25, and this doubled is .5, whose Square Root is .7071, the Side of the Square inscrib'd.

Again, if the Diameter of a Globe be 1, to find the Solidity. In Sect. XI. Chap. II. it is demonstrated, that the Globe is $\frac{2}{3}$ of a Cylinder of the same Diameter and Altitude: Thus, if the Cylinder's Diameter be 1, and its Altitude or Length be also 1, find the Solidity thereof, and take $\frac{2}{3}$ of it, and that will be the Solidity of the Globe requir'd. Now, if the Diameter be 1, the Area of the Circle or Base of the Cylinder, is .7854, (as

(as is above ſhewn) which multiply'd by 1, the Altitude of the Cylinder, and the Product is alſo .7854, the Solidity of the Cylinder; $\frac{2}{3}$ whereof is .5236, which is the Solidity of the Globe, whoſe Diameter is 1.

FROM what has been ſaid, the Reader may eaſily perceive how all other proportional Numbers are found, and may examine them at his Pleaſure.

I ſhall not enlarge any farther upon the Matter, but leave the Book to ſpeak for it ſelf; and if it prove beneficial to the ingenious Practitioners, I have my Deſire. So, wiſhing my ingenious Reader good Succeſs in his Endeavours, not doubting but he will reap Profit hereby; which that he may, is the hearty Deſire of his Well-wiſher.

W. HAWNEY.

THE

COMPLETE MEASURER;

PART. I.

CHAP. I.

Notation of DECIMALS.

A DECIMAL Fraction is an artificial Way of setting down and expressing of Natural or Vulgar Fractions, as whole Numbers: And whereas the Denominators of Vulgar Fractions are divers, the Denominators of Decimal Fractions are always certain: For a Decimal Fraction hath always for its Denominator an Unit, with a Cypher or Cyphers annex'd to it, and must therefore be either 10, 100, 1000, 10000, *&c.* And therefore, in writing down of a Decimal Fraction, there is no Necessity of writing down the Denominator; for by bare Inspection it is certainly known, it consisting of an Unit, with as many Cyphers annexed to it as there are Places (or Figures) in the Numerator.

Example

Example. This Decimal Fraction $\frac{25}{100}$ may be written thus, .25, its Denominator being known to be an Unit with two Cyphers; because there are two Figures in the Numerator. In like Manner, $\frac{125}{1000}$ may be thus written, .125; $\frac{3575}{10000}$ thus, .3575 and; $\frac{75}{1000}$ thus, .075, and $\frac{65}{10000}$ thus, .0065.

As whole Numbers increase in a Decuple, or tenfold Proportion, towards the left Hand, so, on the contrary, Decimals decrease towards the right Hand, in a decuple Proportion, as in the following Scheme.

Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.	Tenth Parts.	Hundredth Parts.	Thousandth Parts.	Ten Thousandth Parts.	Hundred Thousandth Parts.	Millionth Parts, &c.
7	6	5	4	3	2	1	0	1	2	3	4	5	6

Hence it appears, that Cyphers put on the right Hand of whole Numbers, do increase the Value of those Numbers in a decuple (or tenfold) Proportion; but being annexed to the right Hand of a Decimal Fraction, do neither increase nor decrease the Value thereof: So $\frac{2500}{10000}$ is equivalent to $\frac{025}{100}$ or .25. And, on the contrary, tho' in whole Numbers Cyphers prefix'd before them, do neither increase nor diminish the Value; yet Cyphers before a Decimal Fraction, do diminish its Value in a decuple Proportion: For .25, if you prefix a Cypher before it, becomes $\frac{025}{1000}$, or .025; and .125 is $\frac{00125}{100000}$, by prefixing two Cyphers before it, thus, .00125. And therefore, when you are to write a Decimal Fraction, whose Denominator hath more Cyphers than there are Figures in the Numerator, they must be supply'd by prefixing so many Cyphers before the Figures of your Numerator; as, suppose $\frac{19}{1000}$ were to be written down without its Denominator; here, because

cause there are three Cyphers in the Denominator, and but two Figures in the Numerator, therefore prefix a Cypher before 19, and set it down thus, .019.

The Integers are separated from the Decimals several Ways, according to Men's Fancies; but the best and most usual Way, is by a Point or Period; and if there be no whole Number, then a Point before the Fraction is sufficient: Thus, if you were to write down $317\frac{217}{1000}$, it may be thus express'd 317.217; and $59\frac{25}{10000}$ thus, 59.0025; and $\frac{75}{10000}$ thus, .0075, *&c.*

CHAP. II.

Reduction of DECIMALS.

IN *Reduction of Decimals,* there are three Cases: 1st, To reduce a vulgar Fraction to a Decimal. 2dly, To find the Value of a Decimal in the known Parts of Coin, Weights, Measures, *&c.* 3dly, To reduce Coin, Weights, Measures, *&c.* to a Decimal. Of these in their Order.

I. *To reduce a vulgar Fraction to a Decimal.*

The *RULE.*

AS the Denominator of the given Fraction is to its Numerator, so is an Unit (with a competent Number of Cyphers annex'd) to the Decimal requir'd.

Therefore, if to the Numerator given, you annex a competent Number of Cyphers, and divide the Result by the Denominator, the Quotient is the Decimal equivalent to the vulgar Fraction given.

Example

Example 1. Let $\frac{3}{4}$ be given, to be reduc'd to a Decimal of two Places, or having 100 for its Denominator.

To 3 (the Numerator given) annex two Cyphers, and it makes 300; which divide by the Denominator 4, and the Quotient is .75, the Decimal requir'd, and is equivalent to $\frac{3}{4}$ given.

NOTE, That so many Cyphers as you annex to the given Numerator, so many Places must be prick'd off in the Decimal found; and if it shall happen that there are not so many Places of Figures in the Quotient, the Deficiency must be supply'd, by prefixing so many Cyphers before the Quotient-Figures, as in the next Example.

Example 2. Let $\frac{3}{573}$ be reduc'd to a Decimal having six Places.

To the Numerator annex six Cyphers, and divide by the Denominator, and the Quotient is .5235; but it was requir'd to have six Places, therefore you must prefix two Cyphers before it, and then it will be .005235, which is the Decimal requir'd, and is equivalent to $\frac{3}{573}$.

See the Work of these two Examples.

```
4)3.00(.75          573)3.000000(5235
  28                     ....
  ——                    ——————
  20                    1350
  20                     2040
  ——                      3210
  ..                    ——————
                          345
```

In the second Example there remains 345, which Remainder is very insignificant, it being less than $\frac{1}{1000000}$ Part of an Unit, and therefore is rejected.

II. To

II. *To find the Value of a Decimal in the known Parts of Money, Weight, Measures, &c.*

The RULE.

MULTIPLY the Decimal by the Number of Parts in the next inferior Denomination, and from the Product prick off so many Places to the right Hand as there were Places in the Decimal given; and multiply those Figures prick'd off by the Number of Parts in the next inferior Denomination, and prick off so many Places as before, and so continue to do, 'till you have brought it to the lowest Denomination requir'd.

Example 1. Let .7565 of a Pound Sterling be given to be reduc'd to Shillings, Pence, and Farthings.

Multiply by 20, by 12, and 4, as the Rule directs, and always prick off four Places to the right Hand, and you will find it to make 15 *s.* 1 *d.* 2 *q.*

See the WORK.

	.7565
	20
s.	——
	15.1300
	12
d.	——
	1.5600
	4
q.	——
	2.2400

A more compendious Way of finding the Value of the Decimal of a Pound Sterl.

Double the first Figure, (or Place of Primes) and it makes so many Shillings; and if the next Figure (or Place of Seconds) be 5, or more than 5, for the 5 add another Shilling to the

the former Shillings; then for every Unit in the second Place count ten, and to that add the Figure in the third Place, and reckon them so many Farthings; but if they make above 13, abate 1; and if it be above 38, abate 2, and add the remaining Farthings to the Shillings before found.

Example 1. Let .695 of a Pound be reduc'd to Shillings, Pence, and Farthings.

First, Double your 6, and it makes 12 *s.* then take 5 out of 9, and for that reckon another Shilling, and it makes 13 *s.* and the 4 remaining is 4 Tens, and the 5 makes 45, which being above 38, you must therefore cast away 2, and there rest 43 Farthings, which is 10 *d.* $\frac{3}{4}$. So the Answer is 13 *s.* 10 *d.* $\frac{3}{4}$.

	l.		*s.*	*d.*
So the Value of	.725	=	14	6
And the Value of	.878	=	17	6 $\frac{3}{4}$
And the Value of	.417	=	8	4

And so of any other.

Let .53755 of a Pound Troy be reduc'd to Ounces, Pennyweights, and Grains.

Multiply by 12, by 20, and by 24, and always prick off five Places towards the right Hand, and you will find the Answer to be 7 *oz.* 3 *pwt.* 10 *gr. fere.*

See the Work.

```
 .59755
     12
-------
7.17060
     20
-------
3.41200        Facit  oz. pwt. gr.
     24                7   3   9.888
-------
 164800
 82400
-------
9.88800
```

Let

Let .43569 of a Tun be reduc'd to Hundreds, Quarters, and Pounds.

Multiply by 20, by 4, and by 28, and the Answer will be 8 C. 2 *qrs.* 24 *lb. fere.*

.43569	
20	
8.71380	
4	*C. qrs. lb.*
2.85520	*Facit* 8 2 23.9456
28	
23.94560	

Let .9595 of a Foot be reduc'd into Inches and Quarters.

.9595	
12	
11.5140	
4	*Facit* 11 Inches, 2 Quarters.
2.0560	

III. *To reduce the known Parts of Money, Weight, Measure, &c. to a Decimal.*

The *RULE.*

TO the Number of Parts of the lesser Denomination given, annex a competent Number of Cyphers, and divide by the Number of such parts that are contain'd in the greater Denomination, to which the Decimal is to be brought; and the Quotient is the Decimal sought.

Example 1. Let 6 *d.* be reduc'd to the Decimal of a Pound.

To 6 annex a competent Number of Cyphers, (suppose 3) and divide the Result by 240, (the Pence in a Pound) and the Quotient is the Decimal requir'd.

240)6.000(.025

1200

. . . .

Facit ,025

Example 2. Let 3 *d.* $\frac{3}{4}$ be reduc'd to the Decimal of a Pound, having six places.

In 3 *d.* $\frac{3}{4}$ there are fifteen Farthings; therefore to 15 annex six Cyphers, (because there are to be six Places in the Decimal requir'd) and divide by 960, (the Farthings in a Pound) and the Quotient is .015625.

96|0)15.00000|0(.015625

. . . . .

540
600
240
480

. . .

Example 3. Let 3$\frac{1}{4}$ Inches be reduc'd to the Decimal of a Foot, consisting of four Places.

In 3 $\frac{1}{4}$ Inches, there are 13 Quarters? therefore to 13 annex four Cyphers, and divide by 48, (the Quarters in a Foot) and the Quotient is .2708.

48)13.0000(.2708

340
400

16

Example

Example 4. Let 9 *C.* 1 *qr.* 16 *lb.* be reduc'd to the Decimal of a Tun, having six Places.

C. qr. lb.		
9 1 16		224|0)1052.00000'0(.469642.
4		. . . . . .
37 *qrs.*		15600
28		21600
		14400
302	*Facit* .469642	9600
75		6400
1052 Pounds.		1920

CHAP. III.

Addition of DECIMALS.

ADDITION of Decimals is perform'd the same Way as Addition of whole Numbers, only you must observe to place your Numbers right, that is, Units under Units, Primes under Primes, Seconds under Seconds, *&c.*

Example. Let 317.25, 17.125, 275.5, 47.3579, and 12.75, be added together into one Sum.

```
317.25
 17.125
275.5
 47.3579
 12.75
```

Sum 669.9829

This is so plain, that more Examples I think needless.

CHAP. IV.

Subtraction of DECIMALS.

SUBTRACTION of Decimals is perform'd likewise the ſame Way as in whole Numbers, Reſpect being had to the right placing the Numbers, (as in Addition) as in the following Examples.

(1)		(2)	
From	212.0137	From	201.1250
Subtr.	31.1275	Subtr.	5.5785
Reſts	180.8862	Reſts	195.5465
Proof	212.0137	Proof	201.1250

(3)		(4)	
From	2051.315	From	30.5
Subtr.	79.172	Subtr.	7.2597
Reſts	1972.143	Reſts	23.2403
Proof	2051.315	Proof	30.5

NOTE, If the Number of Places in the Decimals be more in that which is to be ſubtracted, than in that which you ſubtract from, you muſt ſuppoſe Cyphers to make up the Number of Places, as in the fourth Example.

CHAP.

CHAP V.

Multiplication of DECIMALS.

MULTIPLICATION of Decimals is also perform'd the same Way as Multiplication of whole Numbers; but to know the Value of the Product, observe this

RULE.

" Cut off, or separate by a Comma, or Prick, so many De-" cimal Places in the Product, as there are Places of Decimals in " both Factors, *viz.* in the Multiplicand and Multiplier, which " I shall farther explain in the following Examples.

Let .3125 be multiply'd by 2.75, multiply the Numbers together, as if they were whole Numbers, and the Product is 8.59375, and because there were three Places of Decimals prick'd off in the Multiplicand, and two Places in the Multiplier, therefore you must prick off five Places of Decimals in the Product, as you may see by the Work.

```
   3.125
    2.75
 -------
   15625
  21875
  6250
 -------
 8.59375
```

Let 79.25 be multiplied by .459.

In this Example, because two Places of Decimals are prick'd in the Multiplicand, and three in the Multiplier, therefore must be five prick'd off in the Product.

```
     79.25
      .459
  --------
     71325
    39625
   31700
  --------
  36.37575
```

Let .135272 be multiply'd by .00425.

In this Example, because in the Multiplicand are six Decimal Places, and in the Multiplier five Places; therefore in the Product there must be eleven Places of Decimals; but when the Multiplication is finish'd, the Product is but 57490600, *viz*, only eight Places; therefore, in this Case, you must prefix three Cyphers before the Product-Figures, to make up the Number of eleven Places; so the true Product will be .00057490600.

```
       .135272
        .00425
  ------------
        676360
       270544
      541088
  ------------
  .00057490600
```

More

More Examples for Practice.

.001472	.017532
.1045	347
7360	122724
5888	70128
1472	52596
.0001538240	6.083604

279.25	32.0752
.445	.0325
139625	1603760
111700	641504
111700	962256
124.26625	1.04244400

4.443	20.0291
15.98	35.45
35544	1001455
39987	801164
22215	1001455
4443	600873
70.99914	710.031595

7.3564	.75432
.0126	.0356
441384	452592
147128	377160
73564	226296
.09269064	.026853792

Con-

Contracted Multiplication of Decimals.

Because in Multiplication of Decimal Parts and mix'd Numbers, there is no Need to express all the Figures of the Product, but in most Cases two, three, or four Places of Decimals will be sufficient; therefore, to contract the Work, observe this following.

RULE.

Write the Unit's Place of the Multiplier under that Place of the Multiplicand, whose Place you intend to keep in the Product; then invert the Order of all the other Figures, that is, write them all the contrary Way. Then, in multiplying, always begin at that Figure in the Multiplicand which stands over the Figure you are then multiplying withal, and set down the first Figure of each particular Product directly one under the other; but yet a due Regard must be had to the Increase arising from the Figures on the right Hand of that Figure in the Multiplicand which you begin to multiply at. This will appear more plain by Examples.

Example 1. Let 2.38645 be multiply'd by 8.2175, and let there be only four Places retain'd in the Decimals of the Product.

First, according to the Directions, write down the Multiplicand, and under it write the Multiplier, thus; place the 8 (being the Unit's Place of the Multiplier) under 4, the fourth Place of Decimals in the Multiplicand, and write the rest of the figures quite contrary to the usual Way, as in the following Work: Then begin to multiply, first the 5 which is left out, (only with Regard to the Increase which must be carry'd from it) saying, 8 times 5 is 40, carry 4 in your Mind, and say 8 times 4 is 32, and 4 I carry, is 36; set down 6 and carry 3, and proceed thro' the rest of the Figures, as in common Multiplication: Then begin to multiply with 2, saying, 2 times 4 is 8, for which I carry 1, (because it is above 5) and say 2 times 6 is 12, and 1 that I carry is 13; set down 3 and carry 1, and proceed thro' the rest of the Figures: Then multiply

multiply with 1, saying, once 6 is 6, for which carry 1, and say once 8 is 8, and 1 is 9; set down 9, and proceed: Then multiply with 7, saying, 7 times 8 is 56, for which carry 6, (because it is above 55) and say 7 times 3 is 21, and 6 that I carry is 27; set down 7 and carry 2, and proceed: Then multiply with 5, saying, 5 times 3 is 15, for which carry 2, and say, 5 times 2 is 10, and 2 I carry is 12, which set down, and add all the Products together, and the total Product will be 19.6107.

See the WORK;

```
     2.38645
     5712.8
    --------
    190916
      4773
       239
       167
        12
    --------
    19.6107
```

NOTE, That in multiplying the Figure left out every Time next the right Hand in the Multiplicand, if the Product be 5, or upwards to 10, you carry 1; and if it be 15, or upwards to 20, carry 2; and if 25, or upwards to 30, carry 3 *&c.*

I have here set down the Work of the last Example wrought by the common Way, by which you may see both the Reason and Excellency of this Way, all the Figures on the right Hand the Line being wholly omitted.

```
          2.38645
          8.2175
    ---------------
        11|93225
       167|0515
       238|645
      4772|90
    190916|0
    ---------------
    19.6106 52875
```

Example 2. Let 375,13758 be multiply'd by 16.7324, so that the Product may have but four Places of Decimals.

First, set 6, the Unit's Place of the Multiplier, under 5, being the fourth Place of Decimals in the Multiplicand, (because four Places of Decimals were to be prick'd off) and write all the rest of the Figures backward. Then multiply all the Figures of the Multiplicand by 1, after the common Way. Then begin with the second Figure of the Multiplier 6, saying 6 times 8 is 48, for which I carry 5, (in respect of the 8 left out) and 6 times 5 is 30, and 5 that I carry is 35; set down 5 and carry 3, and proceed after the common Method. Then begin with 7, the third Figure of the Multiplier, and say, 7 times 5 is 35, for which carry 4, and say, 7 times 7 is 49, and 4 I carry is 53; set down 3 under the first, and carry 5, and proceed as before. Then begin with 3, the fourth Figure of the Multiplier, and say, 3 times 7 is 21, carry 2, and say. 3 times 3 is 9, and 2 I carry is 11; set down 1, and carry 1, and proceed as before. Then begin with 2, the fifth Figure, and say, 2 times 3 is 6, for which I carry 1, and say, 2 times 1 is 2, and 1 I carry is 3; set down 3; and 2 times 5 is 10; set down 0, and carry 1, and proceed as before. Then begin with 4, the last Figure of the Multiplier, and say, 4 times 1 is 4, for which I carry nothing, because 'tis less than 5; then say; 4 times 5 is 20; set down 0. and carry 2, and proceed through the rest of the Figures of the Multiplicand. Then add all up together, and the Product is 6276.9520.

See the WORK.

375,13758 the Multiplicand.
4237.61 the Multiplier revers'd.

37513758 the Product with 1.
22508255 the Product with 6 increas'd with 6 x 8.
2625963 the Product with 7 increas'd with 7 x 5.
112541 the Product with 3 increas'd with 3 x 7.
7503 the Product with 2 increas'd with 2 x 3.
1500 the Product with 4 increas'd with 0.

6276.9520 the Product requir'd.

Let

Let the same Example be repeated, and let only one Place in Decimals be prick'd off.

375.13758 the Multiplicand.
4237.61 the Multiplier inverted.

37514 the Product by 1 with the Increase of 1 x 7.
22508 the Product with 6 increas'd with 6 x 3.
2626 the Product with 7 increas'd with 7 x 1.
113 the Product with 3 increas'd with 3 x 5.
7 the Product with 2 increas'd with 2 x 7.
1 the Increase only of 4 x 3,

6276.9 the Product is the same as before.

More Examples for Practice.

Multiply 395.3756 by .75642, and prick off four Places in Decimals.

395.3756 the Multiplicand.
24657., the Multiplier revers'd.

2767629 the Product by 7 increas'd with 7 x 6.
197688 the Product by 5 increas'd with 5 x 5.
23722 the Product by 6 increas'd with 6 x 7.
1811 the Propuct by 4 increas'd with 4 x 3.
79 the Product by 2 increas'd with 2 x 5.

209.0699 the Product requir'd.

Let

Let the same Example be repeated, and let there be only one Place of Decimals.

395.3756
24657.

1767 the Product by 7 increas'd with 7 x 3.
198 the Product by 5 increas'd with 5 x 5.
24 the Product by 6 increas'd with 6 x 9 + 6 x 5.
2 the Increase of 4 x 8 + 4 x 3.

299.1 the Product.

Characters, and their Signification.

NOTE, That this Mark + signifies Addition; 8 + 5, that is, 8 more 5, or 8 added to 5; and 8 + 3 + 7 denotes these Numbers are to be added into one Sum.

This Mark − signifies Subtraction, as 9 − 4 signifies that 4 is to be taken from 9.

This Mark × signifies Multiplication, as 7 × 5 signifies that 7 is to be Multiply'd into 5.

This Mark ÷ signifies Division, as 12 ÷ 4 signifies 12 is to be divided by 4.

This Mark = signifies Equality, or Equation; that is, when = is plac'd between Numbers or Quantities, it denotes them to be equal, as 7 + 5 = 12, that is 7 more 5 is equal to 12; and 15 − 7 = 8, that is, 15 less by 7, is equal to 8, or subtract 7 from 15 and there remains 8.

This Mark : : is the Sign of Proportion, or the Golden Rule, it being always plac'd betwixt the two middle Terms or Numbers in Proportion, thus, 4 : 20 : : 6 : 30, to be thus read, as 4 is to 20, so is 6 to 30.

CHAP.

CHAP. VI.

Division of DECIMALS.

DIVISION of Decimals is perform'd after the same Manner as Division of whole Numbers; but to know the Value or Denomination of the Quotient, is the only Difficulty, for the resolving of which observe either of the following.

RULES.

I. The first Figure in the Quotient must be of the same Denomination with that Figure in the Dividend which stands (or is to be suppos'd to stand) over the Unit's Place in the Divisor at the first seeking.

II. When the Work of Division is ended, count how many Places of decimal Parts there are in the Dividend more than in the Divisor; for that Excess is the Number of Places which must be separated in the Quotient for Decimals: But if there be not so many Figures in the Quotient, as is the said Excess, that Deficiency must be supply'd with Cyphers in the Quotient, prefix'd before the significant Figures thereof, towards the left Hand, with a Point before them; so shall you plainly discover the Value of the Quotient.

These following Directions ought also to be carefully observ'd.

If the Divisor consists of more Places than the Dividend, there must be a competent Number of Cyphers annex'd to the Dividend, to make it consist of as many (at least) or more Places of Decimals than the Divisor; for the Cyphers added must be reckon'd as Decimals.

Consider whether there be as many decimal Parts in the Dividend as there are in the Divisor; if there be not, make them so many or more, by annexing of Cyphers.

In

In dividing of whole or mix'd Numbers, if there be a Remainder, you may bring down more Cyphers, and, by continuing your Division, carry the Quotient to as many Places of Decimals as you please.

These Things being consider'd, I shall proceed to the Practice of Division of Decimals, which I shall endeavour to explain in as familiar and easy a Method as possible.

Example 1. Let 48 be divided by 144.

In this Example the Divisor 144, is greater than the Dividend 48; therefore, according to the Directions above, I annex a competent Number of Cyphers, (*viz.* four) with a Point between them, and divide according to the usual Way.

144)48.0000(.3333

480
480
480

48

But, first, in seeking how often 144 in 48.0, (the first three Figures of the Dividend) I find the Unit's Place of the Divisor to fall under the first Place of Decimals; therefore the first Figure in the Quotient is in the first Place of Decimals: Or, by the second Rule, there being four Places of Decimals in the Dividend, and none in the Divisor; so the Excess of decimal Places in the Dividend, above that in the Divisor, is four; so that when Division is ended, there must be four Places of Decimals in the Quotient.

See the WORK.

Example 2. Let 217.75 be divided by 65.

First, in seeking how oft 65 in 217, (the first three Figures of the Dividend) I find the Unit's Place of the Divisor to fall under the Unit's Place of the Dividend; therefore the first Figure in the Quotient will be Units, and all the rest Decimals.

Or,

Or, by the ſecond Rule, there being two Places of Decimals in the Dividend, and no Decimals in the Diviſor, therefore the Exceſs of Decimal Places in the Dividend, above the Diviſor, is two; ſo when the Diviſion is ended, ſeparate two Places in the Quotient, towards the right Hand, by a Point.

See the WORK.

65)217.75(3.35

227
325

Example 3. Let 267.15975 be divided by 13.25.

13.25)267.15975(20.163

2159
8347
3975

In this Example, 3, the Unit's Place of the Diviſor, falls under 6. the Ten's Place of the Dividend; therefore (by the firſt Rule) the firſt Figure in the Quotient is Tens: Or, by the ſecond Rule, the Exceſs of Decimal Places in the Dividend, above the Diviſor, is three; there being five Places of Decimals in the Dividend, and but two in the Diviſor, ſo there muſt be three Places of Decimals in the Quotient.

Example 4. Let 15.675159 be divided by 375.89.

375.89)15.675159(.0417

63955
263669

546

In this Example, 5, the Unit's Place of the Divisor, falls under 7, the second Place of Decimals in the Dividend; therefore (by the first Rule) the first Figure in the Quotient is in the second Place of Decimals; so that you must put a Cypher before the first Figure in the Quotient: And by the second Rule, the Excess of Decimal Places in the Dividend, above the Number of Decimal Places in the Divisor, is 4; for the Decimal Places in the Dividend is 6, and the Number of Places in the Divisor but two; therefore there must be four Places of Decimals in the Quotient. But the Division being finish'd after the common Way, the Figures in the Quotient are but three, therefore you must prefix a Cypher before the significant Figures.

Example 5. Let 72.1564 be divided by .1347.

```
.1347)72.1564(535.68
      ------
      4806
       7654
        9190
        11080
        -----
          304
```

In this Example, the Divisor being a Decimal, the first Figure thereof falls under the Ten's Place in the Dividend; therefore the Units (if there had been any) should fall under the Hundred's Place in the Dividend, and so the first Figure in the Quotient is Hundreds. And, by the second Rule, there being four Places of Decimals in the Dividend, and as many in the Divisor, so the Excess is nothing; but in dividing, I put two Cyphers to the Remainders, and continue the Division to two Places farther, so I have two Places of Decimals.

Example

Example 6. Let .125 be divided by .0457.

```
.0457).1250000(2.735
       ....
      ------
       914
      -----
       3360
       3199
      ------
       1610
       1371
      ------
       2390
       2285
      -----
        105
```

In this Example, the Unit's Place of the Divisor (if there had been any) would fall under the Unit's Place of the Dividend; therefore the first Figure of the Quotient is Units. And, by the second Rule, there being seven Places of Decimals in the Dividend, and but four Places in the Divisor, so the Excess is three; therefore there must be three Places of Decimals in the Quotient.

I shall set down only the Work of some few Examples more, and so proceed to *Contracted Division.*

```
.00456).0000059791(.00131
             ..
        ---------
             1419
              511
         --------
               55
```

Let it be divided by 282.

282)1.0000000(.0035461 *fere.*
. . . . .

1540
1300
1720
280

.325).400000(1.2307
. . .

750
1000
2500

225

.042)495.00000(11785.71
. . . . . .

75
330
360
240
300
60

18

Division

Division of DECIMALS *contracted.*

IN Division of Decimals the common Way, when the Divisor hath many Figures, and it is requir'd to continue the Division 'till the Value of the Remainder be but small, the Operation will sometimes be large and tedious, but may be excellently contracted by the following Method.

The *RULE.*

BY the first Rule of this Chapter, (*pag.* 19.) find what is the Value of the first Figure in the Quotient; then, by knowing the first Figure's Denomination, you may have as many or as few Places of Decimals as you please, by taking as many of the left Hand Figures of the Divisor as you think convenient for the first Divisor; and then take as many Figures of the Dividend as will answer them; and in dividing, omit one Figure of the Divisor at each following Operation.

A few Examples will make it plain.

Example 1. Let 721.17562 be divided by 2.25743, and let there be three Places of Decimals in the Quotient.

```
2.25743)721.175|62(319.467
 . . . . .677229
         -------
          43946
          22574
          -----
          21372
          20317
          -----
           1055
            903
           ----
            152
            135
            ---
             17
             15
             --
              2
```

In this Example, the Unit's Place of the Divisor falls under the Hundred's Place in the Dividend; and it is requir'd that three Places of Decimals be in the Quotient; so there must be six Places in all, that is three Places of whole Numbers, and three Places of Decimals. Then, because I can have the Divisor in the first six Figures of the Dividend, I cut off the 62 with a Dash of the Pen, as useless; then I seek how oft the Divisor is in the Dividend, and the Answer is three times; put 3 in the Quotient, and Multiply and Subtract as in common Division, and the Remainder is 43946. Then prick off the 3 in the Divisor, and seek how oft the remaining Figures may be had in 43946, the Remainder, which can be but once; put 1 in the Quotient, and Multiply and Subtract, and the next Remainder is 21372. Then prick off the 4 in the Divisor, and seek how often the remaining Figures may be had in 21372, which will be 9 times; put 9 in the Quotient, Multiply thus; saying, 9 times 4 is 36, for which I carry 4 (in respect of the 4 last prick'd off) and 9 times 7 is 63, and 4 is 67; set down 7 and carry 6, and so proceed 'till the Division be finish'd, always respecting the Increase made from the Figures prick'd off. Observe the Work, which will better inform you than many Words.

2,25743)

```
2.25743)721.17562(319.467

      677229|
     -------|
      43946|6
      22574|3
     -------|--
      21372|32
      20316|87
     -------|---
       1055|450
        902|972
     -------|----
        152|4780
        135|4458
     -------|-----
         17|03220
         15|80201
     -------|------
          1|23019
```

I have set down the Work of this last Example at large, according to the common Way, that thereby the Learner may see the Reason of the Rule, all the Figures on the right Side the perpendicular Line being wholly omitted.

Example 2. Let 5171.59165 be divided by 8.758615, and let it be requir'd that four Places of Decimals be prick'd off in the Quotient.

```
8.758615)5171.5916|5(590.4577
 ......           .
          4379307 5
          ---------
          7922841
          7882754
          ---------
            40087
            35034
            -----
             5053
             4379
             ----
              674
              613
              ---
               61
               61
               --
                ..
```

In this Example, I can't have 8, the first Figure in the Divisor, in 5, the first Figure of the Dividend; so that the Unit's Place of the Divisor falls under the Hundred's Place of the Dividend; so that there will be seven Figures in the Quotient, that is, three of whole Numbers, and four of Decimals; therefore there must be 7 Figures in the Divisor, (because the Number of Places in the Divisor and Quotient will be equal) and there must be eight Places in the Dividend; so that I cut off the Figure 5 with a Dash, as useless. Thus having proportion'd the Dividend to the Divisor, and both to the Number of Places or Figures desir'd in the Quotient, I proceed to divide as before, saying, how often 8 in 51, which will be 5 times; put 5 in the Quotient, and multiply and subtract, and the Remainder is 7922841. Then I prick off the first Figure in the Divisor, 5, and seek how often the remaining Figures in the Divisor, in the aforesaid Remainder, which I find 9 times; put 9 in the Quotient, and multiply thereby, saying, 9 times 5 (the Figure prick'd off) is 45, for which I carry 5, and say, 9 times 1 is 9, and 5 I carry is 14; set down 4, and carry 1, and proceed to multiply the rest of the Figures, and subtract, and the Remainder

der will be 40087. Then prick off the Figure 1, and ſeek how often 87586 in the Remainder 40087, the Anſwer will be 0; ſo put 0 in the Quotient, and prick off the Figure 6, and ſeek how often 8758 in 40087, which will be 4 Times; put 4 in the Quotient, and multiply, ſaying, 4 times 6, (the Figure laſt prick'd off) is 24, for which I carry 2, and ſay, 4 times 8 is 32, and 2 I carry is 34; ſet down 4, and carry 3; multiply the reſt of the Figures, and ſubtract as before, and ſo proceed after the ſame Manner, until all the Figures of the Diviſor be prick'd off, to the laſt Figure.

See the WORK.

Example 3. Let 25.1367 be divided by 217.3543, and let there be five Places of Decimals in the Quotient.

In this Example: 7, the Unit's Place of the Diviſor, falls under 1, the firſt Place of Decimals; therefore the firſt Figure of the Quotient is in the firſt Place of Decimals, ſo the Quotient will be all Decimals: Then, becauſe the Quotient-Figures, and the Figures of the Diviſor, will be of an equal Number, daſh off the 43 in the Diviſor, and the 7 in the Dividend, as uſeleſs, and divide as before.

```
217.35|43)25.136|7(.11564
          21735
          -----
           2401
           2174
           ----
            227
            1087
           ----
            140
            130
            ---
             10
              8
             --
              2
```

Although

Although I have hitherto given Directions for proportioning the Divisor and Dividend, so as to bring into the Quotient what Number of Decimals you please, yet there is no absolute Necessity for it; but you may carry on your Division to what Degree you please, before you begin to prick off the Figures of the Divisor, in order to contract the Work, as in the following Examples, where it is not requir'd to prick off any determinat Number of Decimals, but it may be done according to Discretion.

```
2.756756)7414.76717(2689.67118
 ......       ...
         5513512
         --------
         19012551
         16540536
         --------
         24720157
         22054048
         --------
          2666109
          2481080
          -------
           185029
           165405
           ------
            19624
            19297
            -----
              327
              276
              ---
               51
               28
               --
               23
               22
               --
                1
```

12.34254)

```
12.34254)514.75498(41.705757
          4937016
          -------
          2105338
          1234254
          -------
           871084
           863978
           ------
             7106
             6171
             ----
              935
              864
              ---
               71
               62
               --
                9
                8
                -
                1
```

CHAP. VII.

Extraction of the Square-Root.

If a Square *Number be given.*

To find the Root thereof that is, to find out such a Number, as being multiply'd into it self, the Product shall be equal to the Number given, such Operation is

is call'd, *The Extraction of the* SQUARE ROOT; which to do, obſerve the following Directions.

1*ſt*, You muſt point your given Number, that is, make a Point or Prick over the Unit's Place, another upon the Hundred's, and ſo upon every ſecond Figure throughout.

2*dly*, Then ſeek the greateſt ſquare Number in the firſt Point towards the left Hand, placing the ſquare Number under the firſt Point, and the Root thereof in the Quotient, and ſubtract the ſaid ſquare Number from the firſt Point, and to the Remainder bring down the next Point, and call that the Reſolvend.

3*dly*, Then double the Quotient, and place it, for a Diviſor, on the left Hand of the Reſolvend; and ſeek how often the Diviſor is contain'd in the Reſolvend, (reſerving always the Unit's Place) and put the Anſwer in the Quotient, and alſo on the right Hand Side of the Diviſor; then multiply by the Figure laſt put in the Quotient, and ſubtract the Product from the Reſolvend, (as in common Diviſion) and bring down the next Point to the Remainder, (if there be any more) and proceed as before.

A TABLE *of Squares and Cubes, and their Roots.*

Root	1	2	3	4	5	6	7	8	9
Sqa.	1	4	9	16	25	36	49	64	81
Cub.	1	8	27	64	125	216	343	512	729

Example 1. Let 4489 be a Number given, and let the ſquare Root thereof be requir'd.

$$\begin{array}{r} \dot{4}4\dot{8}9(67 \\ 36 \\ \hline 127)889 \text{ Reſolvend} \\ 889 \text{ Product.} \\ \hline \cdots \end{array}$$

First,

First, Point the given Number, as before directed; then (by the little Table aforegoing) seek the greatest square Number in 44, (the first Point to the left Hand) which you will find to be 36, and 6 the Root; put 36 under 44, and 6 in the Quotient, and subtract 36 from 44, and there remains 8. Then to that 8 bring down the other Point 89, placing it on the right Hand, so it makes 889 for a Resolvend; then double the Quotient 6, and it makes 12, which place on the left Hand for a Divisor, and seek how often 12 in 88, (reserving the Unit's Place) the Answer is 7 times; which put in the Quotient, and also on the right Hand Side of the Divisor, and multiply 127 by 7, (as in common Division) and the Product is 889, which subtracted from the Resolvend, there remains nothing; so is your Work finish'd; and the square Root of 4489 is 67; which Root, if you multiply by it self, that is 67 by 67, the Product will be 4489, equal to the given square Number, and proves the Work to be right.

Example 2. Let 106929 be a Number given, and let the square Root thereof be requir'd.

```
   . . .
  106929(327
  9
  ----
62)169 Resolvend.
   124 Product.
  ----
647)4529 Resolvend.
    4529 Product.
  ----
   ....
```

First, Point your given Number, as before directed, putting a Point upon the Units, Hundreds, and Tens of Thousands; then seek what is the greatest square Number in 10, (the first Point) which by the little Table you will find to be 9, and 3 the Root thereof; put 9 under 10, and 3 in the Quotient; then subtract 9 out of 10, and there remains 1; to which bring down 69, the next Point, and it makes 169 for the Resolvend; then double the Quotient 3, and it makes 6, which place on the left Hand of the Resolvend for a Divisor, and seek how often 6 in 16; the Answer is twice; put 2 in the Quotient, and also on the right Hand

Hand of the Divisor, making it 62. Then multiply 62 by the 2 you put in the Quotient, and the Product is 124; which subtract from the Resolvend, and there remains 45; to which bring down 29, the next Point, and it makes 4529 for a new Resolvend. Then double the Quotient 32, and it makes 64, which place on the left Side the Resolvend for a Divisor, and seek how oft 64 in 452, which you will find 7 times; put 7 in the Quotient, and also on the right Hand of the Divisor, making it 647, which multiplied by the 7 in the Quotient, it makes 4529, which subtracted from the Resolvend, there remains nothing: So 327 is the square Root of the given Number.

Example 3. Let 2268741 be a square Number given, the Root whereof is requir'd.

```
2268741)1506.23
1
-----
25)126
   125
-----
3006)18741
     18036
-----
30122)70500
      60244
-----
301243)1025600
       .903729
-----
Remains .121871
```

Having pointed the given Number as before directed, seek what is the greatest square Number in the first Point 2. which is one; put 1, the Square, under 2, and 1, the Root thereof, in the Quotient; subtract 1 from 2, and there remains 1; to which bring down the next Point, 26, and set on the right Hand, making it 126; double the 1 in the Quotient, which makes 2; set 2 on the left Hand for a Divisor, and ask how oft 2 in 12, which will be 5 times; put 5 in the Quotient, and also on the right Hand of the Divisor, making it 25; multiply (as in

in common Division) 25 by 5, and subtract the Product, 125 from 126, and there remains 1. Bring down the next Point, 87, and it makes 187 for a new Resolvend; and double the 15 in the Quotient, it makes 30 for a new Divisor. Then seek how often 30 in 18, which you can't have; so that you must put 0 in the Quotient, and also on the right Hand of the Divisor, and bring down the next Point, and it makes 18741 for another new Resolvend. Then seek how often 300 in 1874, which will be 6 times; put 6 in the Quotient, and also on the right Hand of the Divisor, multiply and subtract, and the Remainder will be 705. Now, if you have a Mind to find the Value of the Remainder, you may annex Cyphers, by two at a Time, to the Remainders, and so prosecute the Work to what Number of Decimal Parts you please; thus, to 705 annex two Cyphers, and it will make 70500, and the Quotient, doubled, is 3012 for a Divisor: Then seek how often 3012 in 7050 (rejecting the Unit's Place) which will be twice; put 2 in the Quotient, and also on the right Hand of the Divisor, and multiply and subtract as before, and the Remainder will be 10256; to which annex two Cyphers, and proceed as before, and you will get 3 in the Quotient next. So the square Root of the given Number is 1506.23, which being squar'd, or multipli'd, by it self, and the last Remainder added, will make the given Number as follows.

```
                         1506.23
                         1506.23
                        --------
                         451869
                        301246
                       903738
                      753115
                     150623
                    ------------
                    2268728.8129
The Remainder add---------- 12.1871
                    ------------
              Proof 2268741.0000
```

Some

Some more Examples for Practice.

```
                 . . . .
Example 1.   7596796(2756.228 Root.
             4
            ----
           47)359
              329
            ----
          545)3067
              2725
            -----
         5506)34296
              33036
             ------
        55122)126000
              110244
             -------
       551242)1575600
              1102484
             --------
      5512448)47311600
               44099584
              ---------
                3212016
              ---------
```

More Examples for Practice.

```
                  . . . . .
Example 2.   751417.5745(866.84 Root.
             64
            ----
        166)1114
             996
            -----
       1726)11817
             10356
             ------
      17328)146157
             138624
             -------
     173364)753345
             693456
             -------
              59889
             -------
```

If the given Number be a mix'd Number, *viz.* consisting of a whole Number and a Decimal together, make the Number of decimal Places even, that is, 2, 4. 6, 8, &c. that so there may a Point fall upon the Unit's Place of the whole Numbers, as in this last Example, and in that following.

Example. 3. Let 656714.37512 be given, to find the square Root.

```
 . . . . . . .
 656714.375120(810.379 Root.
 64
 ---
 161)167
     161
 ---
 16203)61437
       48609
 ---
 162067)1282851
        1134469
 ---
 1620749)14838220
         14586741
 ---
 Remains 251479
```

In this Example there are five Places of Decimals; therefore put a Cypher to it, to make it even, that so there may a Point fall upon 4, the Unit's Place.

To find the Square Root of a Fraction.

If it be a decimal Fraction, the Work differs nothing from the Examples fore-going, only you must be mindful to point your given Number aright; for (as was before directed) the Number of Places must always be made even, and then begin to point at the right Hand, as in whole Numbers.

If it be a vulgar Fraction, it must be reduc'd to a Decimal, by the first Rule of the second Chapter.

I shall give an Example or two in each Case, and so conclude this Chapter.

Let

Let .125 be a decimal Fraction given, whose square Root is equir'd; and let it be requir'd to have four Places of Decimals in the Root.

```
.12500000(.3535
 9
———
65)350
   325
———
703)2500
    2109
————
7065)39100
     35325
—————
      3775
```

In this Example there must be five Cyphers annex'd, because two Places in the Square make but one in the Root.

Let the square Root of .00715 be requir'd.

```
.007150).084
  64
———
164)750
    656
———
     94
```

In this a Cypher is added to make the Places even.

Let $\frac{7}{8}$ be a vulgar Fraction given, whose square Root is requir'd.

```
                          . . . .
8)7000          (.87500000(.9354
  64              81
  ——              ——
   60       183)650
   56            549
   ——            ———
    40     1865)10100
    40           9325
    ——           ————
    ..    18704)77500
                74816
                —————
                 2684
```

Reduce this $\frac{7}{8}$ to a Decimal, it makes .875 to which annex Cyphers, and extract the square Root, as if it was a whole Number. So the Root is .9354.

Let $\frac{3}{950}$ be a vulgar Fraction, whose square Root is requir'd.

```
                              . . . . . Root.
96|0)3.0000000|0        (.00312500(.0559
       . . . .                25
     288                      ——
     ———                 105)625
     .120                    525
       96                    ———
     ———                1109)10000
      240                     9981
      192                     ————
     ———                        19
      480
      480
     ———
       ..
```

In

In extracting the Root of this, because the first Point consists of Cyphers, there must be a Cypher put first in the Quotient.

To prove this Rule, square the Root, and to the Product add the Remainder, as was before directed. To square a Number, is to multiply it by it self; and to cube it, is to multiply the Square of the Number by the Number it self.

CHAP. VIII.

Extraction of the CUBE-ROOT.

TO extract the CUBE-ROOT, is nothing else but to find such a Number, as being first multiply'd into it self, and then into that Product, produceth the given Number; which to perform, observe these following Directions.

1st, You must point your given Number, beginning with the Unit's Place, and make a Point, or Dot over every third Figure towards the left Hand.

2dly, Seek the greatest Cube Number in the first Point, towards the left Hand, putting the Root thereof in the Quotient, and the said Cube Number under the first Point, and subtract it therefrom, and to the Remainder bring down the Point, and call that the Resolvend.

3dly, Triple the Quotient, and place it under the Resolvend; the Unit's Place of this under the Ten's Place of the Resolvend; and call this the triple Quotient.

4thly, Square the Quotient, and triple the Square, and place it under the triple Quotient; the Units of this under the Ten's Place of the triple Quotient, and call this the triple Square.

5thly, Add these two together, in the same Order as they stand, and the Sum shall be the Divisor.

6thly, Seek how often the Divisor is contain'd in the Resolvend, rejecting the Unit's Place of the Resolvend, (as in the Square Root) and put the Answer in the Quotient.

7thly, Cube the Figure last put in the Quotient, and put the Unit's Place thereof under the Unit's Place of the Resolvend.

8thly, Multiply the Square of the Figure last put in the Quotient into the triple Quotient, and place the Product under the last, one Place more to the left Hand.

9thly, Multiply the triple Square by the Figure last put in the Quotient, and place it under the last, one Place more to the left Hand.

10thly, Add the three last Numbers together, in the same Order as they stand, and call that the Subtrahend.

Lastly, Subtract the Subtrahend from the Resolvend, and if there be another Point, bring it down in the Remainder, and call that a new Resolvend, and proceed in all Respects as before, from the beginning of the third Step to the End of this last.

Example

Example 1. Let 314432 be a Cubick Number, whose Root is requir'd.

314432(68 Root.
216

98432 Resolvend.

18 Triple Quotient of 6.
108 Triple Square of the Quotient 6.

1098 Divisor.

512 Cube of 8, the last Figure of the Root.
1152 The Square of 8. by the triple Quotient.
864 The triple Square of the Quotient 6 by 8.

98432 The Subtrahend.

After you have pointed the given Number, seek what is the greatest Cube Number in 314, the first Point, which, by the former little Table, (*Page* 34) you will find to be 216, which is the nearest that is less than 314, and its Root is 6; which put in the Quotient, and 216 under 314, and subtract it therefrom, and there remains 98; to which bring down the next Point, 432, and annex to 98; so will it make 98432 for the Resolveud. Then triple the Quotient 6, it makes 18, which write down, the Unit's Place, 8, under 3, the Ten's Place of the Resolvend. Then square the Quotient 6, and triple that Square, and it makes 108, which write under the triple Quotient, one Place on the left Hand; then add those two Numbers together, and they make 1098 for the Divisor. Then seek how often the Divisor is contain'd in the Resolvend, (rejecting the Unit's Place thereof) that is, how often 1098 in 9843, which is 8 times; put 8 in the Quotient, and the Cube thereof below the Divisor, the Unit's Place under the Unit's Place of the Resolvend. Then square the 8 last put in the Quotient, and multiply 64, the Square thereof, by the triple Quotient, 18. the Product is 1152: set this under the Cube of 8, the Units of this under the Tens of that.

Then multiply the triple Square of the Quotient by 8, the Figure last put in the Quotient, the Product is 864; set this down under the last Product, a Place more to the left Hand. Then draw a Line under those three, and add them together, and the Sum is 98432, which is call'd the Subtrahend; which being subtracted from the Resolvend, the Remainder is nothing; which shews the Number to be a true cubick Number, whose Root is 68; that is, if 68 be cub'd, it will make 314432.

For, if 68 be multiplied by 68, the Product will be 4624. and this Product, multiplied again by 68, the last Product is 314432, which shews the Work to be right.

	68
	68
	544
The Work.	408
	4624
	68
	36992
	27744
The Proof	314432

Example 2. Let the Cube-Root of 5735339 be requir'd.

After you have pointed the given Number, seek what is the greatest Cube Number in 5, the first Point, which (by the little Table, *pag.* 32) you will find to be 1; which place under 5, and 1, the Root thereof, in the Quotient; and subtract 1 from 5, and there remains 4; to which bring down the next Point, it makes 4735 for the Resolvend. Then triple the 1, and it makes 3; and the Square of 1 is 1, and the Triple thereof is 3; which set one under another, in their Order, and added, makes 33 for the Divisor. Seek how often the Divisor in the Resolvend, and proceed as in the last Example.

5735339

5735339(179 Root.
1

4735 Resolvend.

3 Triple of the Quotient 1, the first Figure.
3 The Triple Square of the Quotient 1.

33 The Divisor.

343 The Cube of 7, the second Figure of the Root.
147 The Square of 7, multiply in the triple Quotient 3.
21 The triple Square of the Quotient, multiply by 7.

3913 The Subtrahend.

822339 The new Resolvend.

51 The triple of the Quotient 17, the two first Figures.
867 The triple Square of the Quotient 17.

8721 Divisor.

729 The Cube of 9, the last Figure of the Root.
4131 The Square of 9, multiply by the triple Quotient 51.
7803 The triple Square of the Quotient 867 by 9.

822339 The Subtrahend.

In this Example, 33, the first Divisor seems to be contain'd more than seven times in 4735, the Resolvend, but if you work with 9, or 8, you will find that the Subtrahend will be greater than the Resolvend.

Some

Some more Examples for Practice.

32461759(319 The Root.
27

5461 Resolvend.

9 The Triple of 3.
27 The triple Square of 3.

279 The Divisor.

1 The Cube of 1, the second Figure.
9 The triple Quotient, by the Square of 1.
27 The triple Square, multiply by 1, the 2d. Figure.

2791 The Subtrahend.

2670759 A new Resolvend.

93 The Triple of 31.
2883 The triple Square of 31.

28923 The Divisor.

729 The Cube of 9, the last Figure.
7533 The Square of 9, by 93, the triple Quotient.
25947 The triple Square 2883 by 9.

2670759 The Subtrahend.

......

84604519(439	The Root.
64	
20604	Resolvend.
12	The Triple of 4.
48	The triple Square of 4.
492	The Divisor.
27	The Cube of 3.
108	The Square of 3, by the triple Quotient.
144	The triple Square by 3.
15507	The Subtrahend.
5097519	The Resolvend.
129	The Triple of 43.
5547	The triple Square of 43.
55599	The Divisor.
729	The Cube of 9.
10449	The Square of 9 by 129.
49923	The triple Square by 9.
5097519	The Subtrahend.
[illegible]	

259697989(638
216

43697 Resolvend.

18 The Triple of 6.
108 The triple Square of 6.

1098 The Divisor.

27 The Cube of 3, the 2d Fignre.
162 The Square of 3 by 18.
324 The triple Square 108 by 3.

34047 The Subtrahend.

9650989 Resolvend.

189 The Triple of 63.
11907 The triple Square of 63.

119259 The Divisor.

512 The Cube of 8.
12096 The Square of 8 by 189.
95256 The triple Square 11907 by 8.

9647072 The Subtrahend.

3917 The Remainder.

22069810125(2805
8

14069 Resolvend.

6 The Triple of 2.
12 The triple Square of 2.

126 The Divisor.

512 The Cube of 8.
384 The Square of 8 by 6.
96 The triple Square by 8.

13952 The Subtrahend.

117810125 New Resolvend.

84 The Triple of 28.
2352 The trip. Square of 28

23604 Divisor.

840 The Triple of 280.
235200 The trip. Squa. of 280.

2352840 New Divisor.

125 Cube of 5.
21000 Square of 5 by 840.
1176000 Triple Square by 5.

117810125 Subtrahend.

....

In this Example 13952, being subtracted from the Resolvend 14069, the Remainder is 117 to which bring down 810, the third Point, and it makes 117810, for a new Resolvend, and the next Divisor is 23604, which you cannot have in the said Resolvend, (the Unit's Place being rejected) so you must put 0 in the Quotient, and seek a new Divisor; (after you have brought down your last Point to the Resolvend;) which new Divisor is 2352840; which you'l find to be contain'd 5 times.

So proceed to finish the rest of the Work.

25917056(295.9	
8	
17917	The Resolvend.
6	The Triple of 2.
12	The triple Square of 2.
126	The Divisor.
729	The Cube of 9, the 2d Figure.
486	The Square of 9 by 6.
108	The triple Square by 9.
16389	The Subtrahend.
1528056	The Resolvend.
87	The Triple of 29.
2523	The triple Square of 29.
25317	The Divisor.
125	The Cube of 5, the 3d Fig.
2175	The Square of 5 by 87.
12615	The triple Square by 5.
1283375	The Subtrahend.
244681000	The Resolvend.
885	The Triple of 295.
261075	The triple Square of 295.
2611635	The Divisor.
729	The Cube of 9, the last Fig.
71685	The Square of 9 by 885.
2349675	The triple Square by 9.
235685079	The Subtrahend.
8995921	The Remainder.

In this Example I annex 3 Cyphers to the Remainder, which make the 3d Resolvend; by which means I bring one Place of Decimals. And so you may proceed to more decimal Places at Pleasure, by annexing 3 Cyphers to the next Remainder, and carrying on the Work as before.

93759.575070(45.42
64

29759 The Resolvend.

12 The Triple of 4, the first Figure.
48 The triple Square of 4.

492 The Divisor.

125 The Cube of 5, the 2d Figure
300 The Square of 5 by 12, the triple Quotient.
240 The triple Square by 5.

27125 The Subtrahend.

2634575 The Resolvend.

135 The Triple of 45.
6075 The triple Square of 45.

60885 The Divisor.

64 The Cube of 4.
2160 The Square of 4 by 135.
24300 The triple Square by 4.

2451664 The Subtrahend.

182911070 The Resolvend.

1362 The Triple of 454.
618348 The triple Square of 454.

6184842 The Divisor.

8. The Cube of 2.
5448 The Square of 2 by 1362.
1236696 The triple Square by 2.

12 724088 The Subtrahend.

59186982 The Remainder.

In

In extracting the Cube Root of a mix'd Number, always observe to make the decimal Parts to consist of either three, six, nine, *&c.* Places, that is, always to consist of even Points, as in the last Example, where the decimal Places were five, to which I annex'd a Cypher to make up six, and so I proceed to point it; and by that Means I have a Point falls upon the Unit's Place of whole Numbers, which you must always observe.

To extract the CUBE-ROOT *out of a Fraction.*

This is the same to do as in whole Numbers, observe but the foregoing Directions for the true pointing thereof; for, as was before directed, the Decimals must always consist of three, six, nine, *&c.* Places; and if it be not so, it must be made so, by annexing of Cyphers, as is abovesaid.

If the CUBE-ROOT of a vulgar Fraction be requir'd, you must first reduce it to a Decimal, and then extract the Root thereof.

Examples of each follow.

Example

Example. 1. Let the Cube Root of 401719179 be requir'd.

401719179 (737	Root.
343	
58719	Resolvend.
21	Triple of 7.
147	Triple Square of 7.
1491	Divisor.
27	Cube of 3.
189	Square of 3 by 21.
441	Triple Square by 3
46017	Subtrahend.
12702179	Resolvend.
219	Triple of 73.
15987	Triple Square of 73.
160089	Divisor.
343	Cube of 7.
10731	Square of 7 by 219.
111909	Triple Square by 7.
11298553	Subtrahend. . . .
1403626	Remainder.

Example 2. Let the Cube Root of .0001418 be requir'd.

```
.000141800(.052 Root.
   125
 -------
 16600 Resolvend.
 -------
    15  The Triple of 5.
    75  Triple Square of 5.
 -------
   765  Divisor.
 -------
     8  Cube of 2.
    60  The Square of 2 by 15.
   150  Triple Square by 2.
 -------
 15608 Subtrahend.
 -------
   992 Remainder.
```

Example 3. Let $\frac{5}{276}$ be a vulgar Fraction whose Cube Root is requir'd.

By the first Rule of Chapter II. Reduce the vulgar Fraction to a Decimal.

```
276)5.0000000000(.018115942
      ....
    -----
    2240
      320
       440
       1640
        2600
         1160
          560
          ---
            8
```

.018115942(.262 Root.
8

10115 Resolvend.

6 Triple of 2.
12 Triple Square of 2.

126 Divisor.

216 Cube of 6.
216 Square of 6 by the Triple of 2.
72 The triple Square by 6.

9576 Subtrahend.

539942 Resolvend.

78 Triple of 26
2028 Triple Square of 26.

20358 Divisor.

8 Cube of 2.
312 Square of 2 by 78.
4056 Triple Square 2028 by 2.

408728 Subtrahend.

131214 Remainder.

You may prove the Truth of the Work, by Cubing the Root found, as was shew'd in the first Example; and if any Thing remains, add it to the said Cube, and the Sum will be the given Number, if the Work is rightly perform'd.

I will shew the Proof of the fifth Example, (*Pag.* 48) the given Number being 259697989, whose Root is 638, it being a surd Number there remains 3917.

	638
	638
	5104
	1914
	3828
The Square	407044
	638
	3256352
	1221132
	2442264
The Cube	259694072
The Remainder add	3917
Proof equal to the given Number	259697989

CHAP. IX.

Multiplication of Feet, Inches, and Parts

IN the Multiplying of Feet, Inches, &c. I shall endeavour to lay down such easy and familiar Rules, as may easily be understood by the meanest Capacity.

Exam-

Example 1. Let 7 Feet 9 Inches be multiply'd by 3 Feet 6 Inches.

F.	I.	
7	9	
3	6	
23	3	*Pts.*
3	10	6
27	1	6

Firſt, Multiply 9 Inches by 3, ſaying, 3 times 9 is 27 Inches, which make 2 Feet 3 Inches; ſet down 3 under Inches, and carry 2 to the Feet, ſaying, 3 times 7 is 21, and 2 that I carry make 23; ſet down 23 under the Feet.

Then begin with 6 Inches, ſaying, 6 times 9 is 54 Parts, which is 4 Inches and 6 Parts; ſet down 6 Parts, and carry 4, ſaying, 6 times 7 is 42, and 4 that I carry, is 46 Inches, which is 3 Feet 10 Inches; which ſet down, and add all up together, and the Product is 27 Feet 1 Inch 6 Parts.

Example 2. Let 75 Feet 7 Inches be multiply'd by 9 Feet 8 Inches.

F.	I.	
75	7	
9	8	
680	3	
50	4	8
730	7	8

Firſt, Multiply by 9 Feet, ſaying, 9 times 7 is 63, which is 5 Feet 3 Inches; ſet down 3 and carry 5 ſaying, 9 times 5 is 45, and 5 I carry is 50; ſet down 0 and carry 5, ſaying, 9 times

7 is 63, and 5 is 68; set down 68, and proceed to multiply by 8 Inches, saying, 8 times 7 is 56, the Twelves in 56 are 4 times, and 8 remains; set 8 a Place to the right Hand and carry 4: Then multiply 75 by 8, and the Product is 600, and 4 that I carry is 604, which divided by 12, the Quotient is 50 Feet, and 4 Inches, and add all up together, and you will find the Product 730 Feet, 7 Inches, 8 Parts.

I will repeat the last Example again, and shew another Way to work it, which I think is better, and more expeditious, when there are more Figures than one in the Feet; thus.

F.	I.		
75	7		
9	8		
680	3		
25	2	4	$\frac{1}{3}$
25	2	4	$\frac{1}{3}$
730	7	8	

Multiply by 9 Feet, first, as above directed; then, instead of multiplying by 8 Inches, let the 8 Inches be parted into such aliquot or even Parts of a Foot, as you find to be contain'd in that Figure; if you take such Parts of the Multiplicand, and add them to the former Product, the Sum will give the Answer: Thus, 8 Inches may be parted into 4 and 4, because 4 is the third Part of 12. So, if you take the third Part of 75 Feet 7 Inches, and set it down twice, and add all together, the Sum will be 730 Feet 7 Inches 8 Parts, the same as before; thus, say how often 3 in 7, which is twice, set down 2; then, because twice 3 is 6, say, 6 out of 7 and there remains 1, for which you must add 10 to the 5, and it makes 15; then the Threes in 15 are 5 times, set down 5; and, because 3 times 5 is 15, there is 0 remains. Then go to the 7 Inches, saying, the Threes in 7 are twice; set down 2 in the Inches; and because twice 3 is but 6, take 6 out of 7, and there remains 1 Inch, which is 12 Parts; then the Threes in 12 are 4 times, and 0 remains. So the third Part of 75 Feet 7 Inches, is 25 Feet, 2 Inches, 4 Parts, which set down again, and add all together, the Sum is 730 Feet, 7 Inches, 8 Parts, the same as before.

Example

Example 3. Let 97 Feet 8 Inches be multiply'd by 8 Feet 9 Inches.

F.	I.	
97	8	$\frac{1}{2}$
8	9	$\frac{1}{4}$
781	4	
48	10	
24	5	
854	7	

Begin, first, to multiply by 8 Feet, saying, 8 times 8 is 64 Inches, that is 5 Feet 4 Inches; set down 4 Inches, and carry 5, saying, 8 times 7 is 56, and 5 I carry is 61; set down 1, and carry 6, saying 8 times 9 is 72, and 6 I carry is 78, which set down: Then, instead of Multiplying by 9 Inches, take the aliquot Parts of 12, which 9 makes, which is 6 and 3, 6 Inches being half 12. and 3 the fourth Part; therefore take the half of 97 Feet 8 Inches, which is 48 Feet 10 Inches, and because 3 is half 6, you may take the half of 48 Feet 10 Inches, which is 24 Feet 5 Inches; add all up together, and the Sum is 854 Feet 7 Inches.

See the WORK, as above.

Example 4. Let 75 Feet 9 Inches be multiply'd by 17 Feet 7 Inches.

F.	I.	
75	9	
17	7	
525		
75		
25	3	P
18	11	3
8	6	
4	3	
1331	11	3

In this Example, because there are more than 12 Feet in the Multiplier, therefore I first multiply the 75 by 17 Feet; then because the aliquot Parts in 7 Inches are 4 and 3, that is, a third and a fourth, I take the third Part of 75 Feet 9 Inches, which is 25 Feet 3 Inches, and the fourth Part thereof is 18 Feet 11 Inches

 3 Parts

3 Parts; then the aliquot Parts of 9 Inches are 6 and 3, that is, half and a fourth; therefore I take half 17 Feet, which is 8 Feet 6 Inches, and the fourth Part is 4 Feet 3 Inches, (not meddling with tee 7 Inches because that was multipli'd into the 9 before;) then add all these together, and the Sum is 1331 Feet, 11 Inches, 3 Parts.

Example 5. Let 87 Feet 5 Inches be multiply'd by 35 Feet 8 Inches.

F.	I.		
87	5		
35	8		
435			
261		P	
29	1	8	$\frac{1}{3}$
29	1	8	$\frac{1}{3}$
11	8	0	$\frac{1}{3}$
2	11	0	$\frac{1}{12}$
3117	10	4	

Work here as in the last Example. After you have Multiply'd the Feet, then take the aliquot Parts of 8 Inches, which is two Thirds; therefore take the third Part of 87 Feet 5 Inches and set it down twice. Thus, the third Part of 87 Feet 5 Inches, is 29 Feet, 1 Inch, 8 Parts; set this down twice: Then the aliquot Parts of 5 Inches are 4 and 1, that is, a third Part and a twelfth Part; therefore take a third Part of 35, which is 11 Feet 8 Inches, and a twelfth Part of 35, is 2 Feet 11 Inches; set all these one under another, and add them together, and the Sum is 3117 Feet, 10 Inches, 4 Parts.

Example 6. Let 259 Feet 2 Inches be multiply'd by 48 Feet 11 Inches.

F.	I.		
259	2		
48	11		
20072			
1036			
129	7	P	$\frac{1}{2}$
86	4	8	$\frac{1}{3}$
21	7	2	$\frac{1}{12}$
8	1	10	$\frac{1}{6}$
42677	7	8	

First,

First, Multiply the Feet; then take the aliquot Parts of 11, which will be 6, 4, and 1, that is, an half, a third, and a twelfth; therefore take the half of 259 Feet 2 Inches, which is 129 Feet 7 Inches; and a third Part is 86 Feet, 4 Inches, 8 Parts; and the twelfth Part of 259 Feet 2 Inches, is 21 Feet, 7 Inches, 2 Parts; or (because 1 is the fourth Part of 4) you may more readily take the fourth Part of 86 Feet, 4 Inches, 8 Parts, which is also 21 Feet, 7 Inches, 2 Parts; then add all together, and the Sum is 12677 Feet, 7 Inches, 8 Parts.

See the foregoing WORK.

I shall set down only the working of some few Examples in Feet and Inches, and then proceed to Multiply Feet, Inches, and Parts, &c.

```
          F.  I.
         179  3
          38 10
        ---------
        1432
         537      P
          89  7  6 | 1/2
          59  9  0 | 1/3
           9  8  6 | 1/4
        ---------
Product 6961  1  0
```

```
          F.  I.
         246  7
          46  4
        ---------
        1476
         984      P
          82  2  4 | 1/2
          15  5  4 | 1/3
          11  7  0 | 1/4
        ---------
Product 11425  2  8
```

```
          F.  I.
         246  7
          36  9
        ---------
        1476
         738      P
         123  3  6 | 1/2
          61  7  9 | 1/4
          12  3  0 | 1/3
           9  2    | 1/4
        ---------
Prod[illegible] 9062  4  7
```

```
          F.  I.
         257  9
          39 11
        ---------
        2313
         771      P
         128 10  6 | 1/2
          85 11  0 | 1/3
          21  5  9 | 1/12
          19 11  6 | 1/4
           9 11  9 | 1/4
        ---------
Product 10289  2  6
```

Example

Example 11. Let 7 Feet, 5 Inches, 9 Parts, be multiply'd by 3 Feet, 5 Inches, 3 Parts.

F.	I.	P.		
7	5	9		
3	5	3		
22	5	3	S	
3	1	4	9	T
	1	10	5	3
25	8	6	2	3

In this Example, I first begin with 3 Feet, and thereby multiply 7 Feet, 5 Inches, 9 Parts: First, I say, 3 times 9 is 27 Parts, that is, 2 Inches and 3 Parts; set down 3 under the Parts, and carry 2, saying, 3 times 5 is 15, and 2 I carry is 17, that is, 1 Foot 5 Inches; set down 5 Inches, and carry 1, and say, 3 times 7 is 21, and 1 I carry is 22; set down 22 Feet. Then begin with 5 Inches, saying, 5 times 9 is 45, which is 45 Seconds, which make 3 Parts and 9 Seconds; set down 9 Seconds, a Place towards the right Hand, and carry 3 Parts, saying, 5 times 5 is 25, and 3 I carry is 28, which is 2 Inches and 4 Parts; set down 4 Parts, and carry 2, saying, 5 times 7 is 35, and 2 I carry is 37, which is 3 Feet 1 Inch; set down 3 Feet 1 Inch, and begin to multiply by 3 Parts, saying, 3 times 9 is 27 Thirds, that is 2 Seconds and 3 Thirds; set down 3 Thirds, and carry 2, saying, 3 times 5 is 15, and 2 I carry is 17, that is, 1 Part and 5 Seconds; set down 5 Seconds, and carry 1, saying, 3 times 7 is 21, and 1 I carry is 22, which is 1 Inch and 10 Parts, which set down, and add all up, and the Product is 25 Feet, 8 Inches, 6 Parts, 2 Seconds, 3 Thirds.

NOTE, That in Multiplying Feet, Inches, and Parts, &c. if Feet be multiply'd by Feet, the Product is Feet; and Feet multiply'd by Inches, the Product is Inches; and the twelfth Part is Feet; and Parts multiply'd by Feet, the Product is Parts, and the twelfth Part thereof is Inches; Parts multiply'd by Inches, the Product is Seconds, and the twelfth Part thereof is Parts; and Parts multiplied by Parts the Product is Thirds, and the twelfth Part thereof is Seconds.

So that if you begin to multiply Parts by Feet in the first Row, and Parts

Parts by Inches in the second Row, and Parts by Parts in the third Row, the first Figure in every Row will stand a Place more towards the right Hand, as you may see in the last Example.

Example 12. Let 37 Feet, 7 Inches, 5 Parts, be multiply'd by 4 Feet, 8 Inches, 6 Parts.

F.	I.	P			
37	7	5.			
4	8	6			
150	5	8	S		
12	6	5	8		$\frac{1}{3}$
12	6	5	8	T	$\frac{1}{3}$
1	6	9	8	6	$\frac{1}{2}$
177	1	5	0	6	

First, I multiply by 4 Feet, saying, 4 times 5 is 20, which is 1 Inch, 8 Parts; set down 8, and carry 1, saying 4 times 7 is 28, and 1 I carry is 29, which is 2 Feet, 5 Inches; set down 5 Inches, and carry 2, saying, 4 times 7 is 28, and 2 I carry is 30; set down 0, and carry 3, and say, 4 times 3 is 12, and 3 is 15; set down 15. Then I begin with 8 Inches; but, because the Feet in the Multiplicand are more than 12, it will be the best Way to work for the aliquot Parts of 8; so here I work for 4 Inches, and set that down twice, 4 being the third Part of 12; therefore take the third Part of 37 Feet, 7 Inches, 5 Parts, which is 12 Feet, 6 Inches, 5 Parts, 8 Seconds; set this down twice. Then begin with 6 Parts; but, instead of multiplying, take half 37 Feet, 7 Inches, 5 Parts (because 6 is half 12) and set it a Place more to the right Hand: Thus, the half of 37 Feet is 18, which I must count 18 Inches, because the Multiplier is 6 Parts; so the half of 37 Feet, 7 Inches, 5 Parts, is 1 Foot, 6 Inches, 9 Parts, 8 Seconds, 6 Thirds; which set down, and add all up together, and the Sum is 177 Feet, 1 Inch, 5 Parts, 0 Seconds, 6 Thirds.

Example 13. Let 311 Feet, 4 Inches, 7 Parts, be multiply'd by 36 Feet, 7 Inches, 5 Parts.

F.	I.	P.	S	T	
311	4	7			
36	7	5			
1866					
933					
103	9	6	4		$\frac{1}{3}$
77	10	1	9		$\frac{1}{4}$
8	7	9	6	4	$\frac{1}{3}$
2	1	11	4	7	$\frac{1}{4}$
12	0	0	0	0	$\frac{1}{3}$
1	0	0	0	0	$\frac{1}{12}$
	9	0	0	0	$\frac{1}{3}$
11402	2	4	11	11	

In this Example, because the Feet both in the Multiplier and Multiplicand are compound Numbers, I first multiply the Feet one by the other; then take the aliquot Parts of 7 Inches, which are 4 Inches and 3, that is, a third and a fourth Part; so take the third Part of 311 Feet, 4 Inches, 7 Parts, which is 103 F. 9 I. 6 P. 4 S. and the fourth Part is 77 F. 10 I. 1. P. 9 Sec. set these down one under another, the Feet under the other Feet; then the aliquot Parts of 5 Parts are 4 and 1, that is, a third and twelfth Part; so the third Part of 311 Feet, 4 Inches, 7 Parts, is 103 Feet, 9 Inches, 6 Parts, 4 Seconds, but because the Multiplier is Parts, it must be set a Place to the right Hand, that is, the 103 must be Inches, which is 8 Feet 7 Inches; therefore I set down 8 Feet, 7 Inches, 9 Parts, 6 Seconds, 4 Thirds.

Then, because 1 Inch is a fourth Part of 4 Inches, therefore I take a fourth Part of 8 Feet, 7 Inches, 9 Parts, 6 Seconds, 4 Thirds, which is 2 Feet, 1 I. 11 Parts 4 Seconds 7 Thirds, which is the same as if I had taken a twelfth Part of 311 Feet, 4 Inches, 7 Parts,

7 Parts. Then, for 4 Inches in the Multiplicand, instead of multiplying 36 Feet by it, take a third Part, because 4 Inches is a third Part of 12; so the third Part of 36 is 12 Feet, and the aliquot Parts of 7 Parts, are 4 and 3, that is, a third and a fourth; so the third Part of 36 is 12, which now is 12 Inches, that is, 1 Foot, and the fourth Part is 9 Inches; add all these together, and the Sum will be 11402 Feet, Inches, 4 Parts, 11 Seconds, 11 Thirds.

Example 14. Let 8 Feet, 4 Inches, 3 Parts, 5 Seconds, 6 Thirds, be Multiply'd by 3 Feet, 3 Inches, 7 Parts, 8 Seconds, 2 Thirds.

	F.	I.	P.	S.	T.				
	8	4	3	5	6				
	3	3	7	8	2				
	25	0	10	4	6				
	2	1	0	10	4	6			
		4	10	6	0	2	6		
			5	6	10	3	8	0	
				1	4	8	6	11	0
Product	27	7	3	5	1	8	8	11	0

In this last Example there is no Difficulty, if you do but observe the former Directions, and set every Row a Place more to the right Hand.

I shall

I shall only set down the Working of some few Examples more, and so conclude this Chapter.

F.	I.	P.		
321	7	3		
9	3	6		
2894	5	3	S	
80	4	9	9	T
13	4	9	7	6
2988	2	10	4	6

F.	I.	P.	
42	7	8	
7	3	6	
298	5	8	
10	7	11	S
1	9	3	10
310	10	10	10

F.	I.	P.		
124	7	9		
14	6	2		
496				
124			S	
62	3	10	6	T
1	8	9	3	6
7	0	0	0	0
1	2	0	0	0
	7	0	0	0
	3	6	0	0
1809	1	1	9	6

F.	I.	P.		
259	10	8		
18	5	4		
2072				
259			S	
86	7	6	8	
21	7	10	8	T
7	2	7	6	8
9	0	0	0	0
6	0	0	0	0
1	0	0	0	0
4793	6	0	10	8

F.	I.	P.			
267	7	10			
25	9	7			
1335					
534					
133	9	11	S		
66	10	11	6		
11	1	9	11	T	
1	10	3	7	10	
12	6	0	0	0	
2	1	0	0	0	
1	0	6	0	0	
	8	1	0	0	
6905	0	7	0	10	

F.	I.	P.			
317	9	7			
37	5	9			
2219					
951			S		
105	11	2	4		
26	5	9	7	T	
13	2	10	9	6	
6	7	5	4	9	
18	6	0	0	0	
9	3	0	0	0	
1	6	6	0	0	
	3	1	0	0	
11910	9	11	1	3	

RULE

The R U L E *at Large.*

Feet Multiplied by	*Feet* *Inches* *Parts*	*produce*	*Feet* *Inches* *Parts*
Inches Multipli'd by	*Feet* *Inches* *Parts*	*produce*	*Inches* *Parts* *Secon.*
Parts Multipli'd by	*Feet* *Inches* *Parts*	*produce*	*Parts* *Secon.* *Thirds*

N. B. 12 *Thirds make one Second,* 12 *Seconds make one Part,* 12 *Parts make one Inch, and* 12 *Inches make one Foot in this Multiplication of Feet, Inches and Parts.*

THE COMPLETE MEASURER;

PART II.

CHAP. I.

Mensuration of SUPERFICIES.

Superficial Figures are all such as have only Length and Breadth, not having any commensurable Thickness.

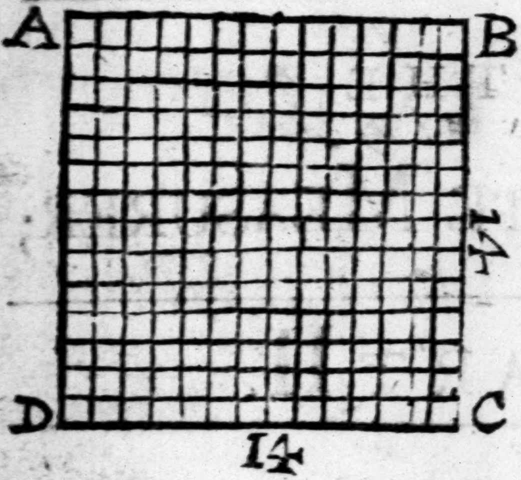
§ I. *Of a* SQUARE.

A SQUARE is a Geometrical Figure, having four equal Sides, and as many right (or square) Angles. To find the superficial Content thereof, this is

The RULE.

Multiply the Side into it ſelf, and the Product is the Content.

Let ABCD be a Geometrical Square given, each Side being 14 Feet, Yards, Poles, or other Meaſure; multiply 14 by it ſelf, and the Product is 196, which is the ſuperficial Content.



14
14
56
14
196 Product.

By Scale and Compaſſes.

Extend the Compaſſes from 1, in the Line of Numbers, to 14 the ſame Extent will reach from the ſame Point, turn'd forward to 196.

DEMONSTRATION.

Let each Side of the given Square be divided into 14 equal Parts, and Lines drawn from one another, croſſing each other within the Square; ſo ſhall the whole great Square be divided into 196 little Squares, as you may ſee in the Figure above, equal to the Number of ſquare Feet, Yards, or Poles, or other Meaſures, by which the Side was meaſur'd.

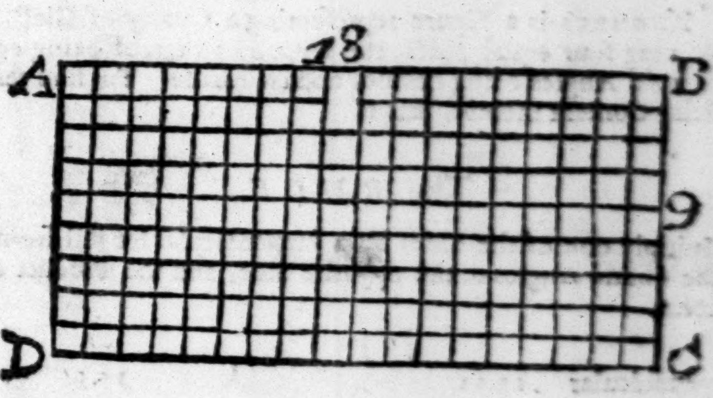
§ II. Of

§ II. *Of a* PARALLELOGRAM, *or* LONG-SQUARE.

A Parallelogram is a Figure having four Sides, and as many right Angles, the opposite Sides thereof being equal and parallel. To find the superficial Content thereof, this is

The *RULE.*

Multiply the Length by the Breadth, and the Product is the superficial Content.



Length	18
Breadth	9
Product	162

Let ABCD be a long Square, the Length thereof 18 Feet, and the Breadth 9 Feet, which multiply'd together, the Product is 162, the superficial Content thereof.

By Scale *and* Compasses.

Extend the Compasses in the Line of Numbers from 1 to 9, the same Extent will reach from 18 down to 162, the Square Feet.

DEMONSTRATION.

If the Sides, AB and CD, be each divided into 18 equal Parts, representing 18 Feet; and the Lines AD and BC each divided into 9 equal Parts, and Lines drawn from Point to Point, crossing each other within the Figure; those Lines will make thereby so many little Squares as there are square Feet, *viz.* 162.

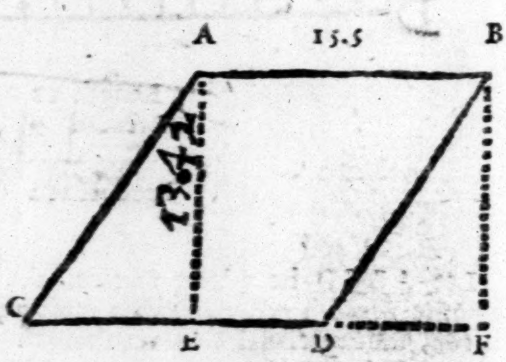
§ III. *Of a* RHOMBUS.

A RHOMBUS is a Figure representing a Quarry of Glass, having four equal Sides, the Opposites thereof being equal, two Angles being obtuse, and two acute. To find the superficial Content thereof, this is

The *RULE.*

Multiply one of the Sides by a Perpendicular let fall from one of the obtuse Angles to the opposite Side, and the Product is the Content.

Perpendidcular	13.42
The Side	15.5
	6710
	6710
	1342
Product	208.010



Let ABCD be a Rhombus given, whose Sides are each 15.5 Feet, and the Perpendicular EA is 13.42, which multiply'd together, the Product is 208.01; which is the superficial Content of the Rhombus, that is, 208 Feet and one hundredth Part of a Foot.

By

By Scale and Compasses.

Extend the Compasses from 1 to 13.42, that Extent will reach from 15.5, the same Way to 208 Feet, the Content.

DEMONSTRATION.

Let C D be extended out to F, making D F equal to C E, and draw the Line B F; so shall the Triangle D B F be equal to the Triangle A C E: For D F and C E are equal, and B F is equal to A E, because A B and C F are parallel. Therefore the Parellelogram A B E F is equal to the Rhombus A B C D.

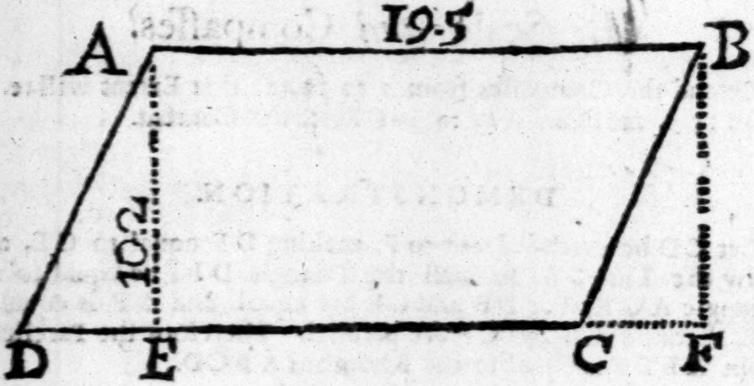
§ IV. Of a RHOMBOIDES.

A Rhomboides is a Figure having four Sides, the opposite whereof are equal and parallel; and also four Angles, the opposite whereof are equal. To find the superficial Content thereof, this is

The RULE.

Multiply one of the longest Sides thereof by the Perpendicular let fall from one of the obtuse Angles to one of the longest Sides, and the Product is the Content.

$$
\begin{array}{r}
19.5 \\
10.2 \\
\hline
390 \\
1950 \\
\hline
198.90
\end{array}
$$



Let ABCD be a Rhomboides given, whose longest Sides, AB or CD, is 19.5 Feet, and the Perpendicular AE is 10.2; which multiply'd together, the Product is 198,9, that is, 198 superficial Feet, and 9 tenth Parts, the Content.

DEMONSTRATION.

If DC be extended to F, making CF equal to DE, and a Line drawn from B to F; so will the Triangle CBF be equal to the Triangle ADE, and the Parallelogram AEFB be equal to the Rhomboides ABCD, which was to be prov'd.

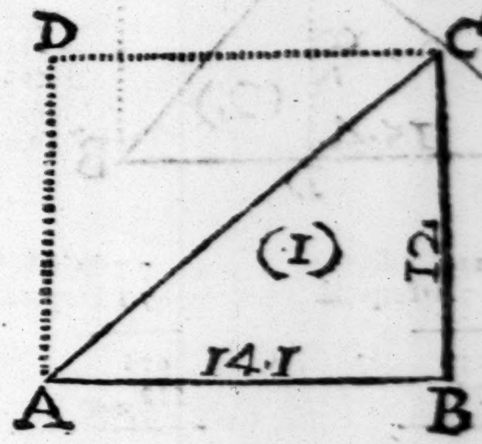
§ V. *Of a* TRIANGLE.

A Triangle is a Figure having three Sides and three Angles. Triangles are either right-angled or oblique-angled. Right-angled Triangles are such as have one right-Angle. Oblique-angled Triangles are such as have their Angles either acute or obtuse. An obtuse Angle is greater than a right Angle, that is, it is more than 90 Degrees; and an acute Angle is less than a right Angle. To find the superficial Content thereof, this is

The

The RULE.

Let the Triangle be of what kind ſoever. Multiply the Baſe by half the Perpendicular, or half the Baſe by the whole Perpendicular; or multiply the whole Baſe by the whole Perpendicular; and take half the Product; any of theſe three Ways will give the Content.

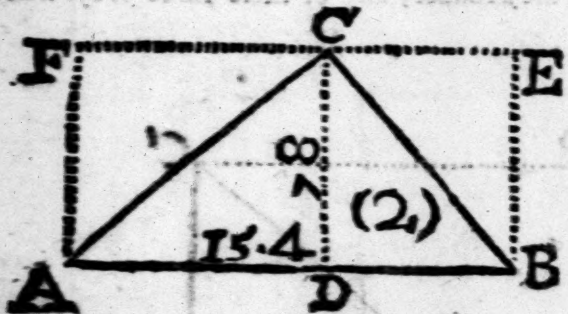


Let A B C be a right-angled Triangle, whoſe Baſe is 14.1 Feet, and the Perpendicular 12 Feet, Multiply 14.1 by 6 half the Perpendicular, and the Product is 84.6 Feet, the Content. Or multiply 14.1 by 12, the Product is 169.2; the half thereof is 84.6, the ſame as before.

14.1 Baſe.	14.1 Baſe.
6 Half Perpendicular.	12 Perpendicular;
84.6 Product.	169.2 Product.
	84.6 Half.

By Scale *and* Compaſſes.

Extend the Compaſſes from 2 to 14.1, that Extent will reach the ſame Way from 12 to 84.6 Feet, the Content.



15.4 Baſe.
3.9 half perpend.

1386
462

60.06 Product.

15.4 Baſe.
7.8 Perpend.

1232
1078

120.12

60.06 Half.

7.7 half Baſe.
7.8 Perpend.

616
539

60.06 Product.

Let A B C (*Fig.* 2.) be an oblique-angled Triangle given, whoſe Baſe is 15,4, and the Perpendicular 7.8; if 15.4 be multiply'd by 3.9, (half the Perpendicular) the Product will be 60.06 for the Area or ſuperficial Content: Or if the Perpendicular 7.8 be multiply'd into half the Baſe 7.7, the Product will be 60.06, as before: Or if 15.4, the Baſe, be multiply'd by the whole Perpendicular 7.8, the Product will be 120.12, which is the double Area; the Half thereof is 60.06 Feet, as before.

See

See the WORK.

By Scale *and* Compasses.

Extend the Compasses from 2 to 15.4, that Extent will reach from 7.8 to 60 Feet, the Content.

DEMONSTRATION.

If AD (*Fig.* 1.) be drawn parallel to BC, and DC parallel to AB; the Triangle ADC shall be equal to the given Triangle ABC. Hence the Parallelogram ABCD is double to the given Triangle; therefore half the Area of the Parallelogram is the Area of the Triangle. In *Fig.* 2. the Parallelogram ABEF is also double to the Triangle ABC; for the Triangle ACF is equal to the Triangle ACD, and the Triangle BCE is equal to the Triangle BCD; therefore the Area of the Parallelogram is double to the Area of the given Triangle. Which was to be prov'd.

To find the Area of any plain Triangle, by having the three Sides given, without the Help of a Perpendicular.

The RULE.

Add the three Sides together, and take half that Sum; then subtract each Side severally from that half Sum. Which done, multiply that half Sum and the three Differences continually, and out of the last Product extract the square Root, which square Root shall be the Area of the Triangle sought.

Example

Example. Let A B C be a Triangle, whofe three Sides are as followeth, *viz.* A B 43.3, A C 20.5, and B C 31.2, the Area is requir'd.

	Sides	Differences
	43.3	4.2
	31.2	16.3
	20.5	27.0
Sum	95.0	
Half	47.5	

Area

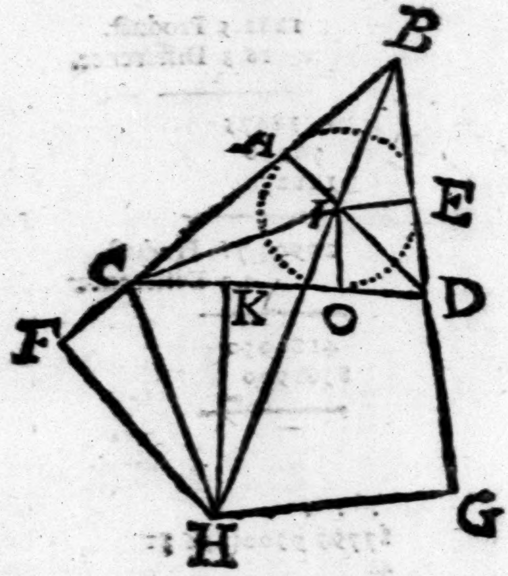
Area 296.31

```
    47.5 The half Sum.
    27   Difference.
    ----
    3325
    950
    ----
    1282.5 Product.
      16.3 Difference.
    ------
     38475
    76950
   12825
   --------
   20904.75 Product.
        4.2 Difference.
   --------
   4180950
  8361900
  --------
  87799.9500(296.31
  4
  --
49)477
   441
   ---
586)3699
    3516
    ----
5923)18395
     17769
     -----
59261)62600
      59261
      -----
       3339 Remains.
```

DEMON.

DEMONSTRATION.

In the Triangle BCD, I ſay, if from the half Sum of the Sides, you ſubtract each particular Side, and multiply the half Sum and the three Differences together continually, the ſquare Root of the Product ſhall be the Area of the Triangle.



First, by the Lines BI, CI, and DI, biſſect the three Angles, which Lines will all meet in the Point I; by which Lines the given Triangle is divided into three new Triangles, CBI, DCI, and BDI; the Perpendiculars of which new Triangles, are the Lines AI, EI, and OI, being all equal to one another, becauſe the Point I is the Center of the inſcrib'd Circle, (by *Euclid, Lib.* 4. *Prop.* 4.) Wherefore to the Side BC join CF equal to DE, or DO; ſo ſhall BF be equal to half the Sum of the Sides, *viz.* $= \frac{1}{2}BC + \frac{1}{2}BD + \frac{1}{2}CD$.

And $BA = BF - CD$ for $CA = CO$ and $OD = CF$, therefore $CD = AF$, and $AC = BF - BD$ for $BE = BA$ and $ED = CF$, therefore, $BD = BA + CF$, and $CF = BF - BC$.

Then make $CK = CF$, and draw the Perpendiculars, FH, GH, and KH, and extend BI to H; becauſe the Angles $FCK + FHK$ are equal to two right Angles, (for the Angles F and K are right Angles) equal alſo to $FCK + ACO$, (by *Euclid* 1. 13)

And

And the Angles ACO + AIO are equal to two right Angles; therefore the Quadrangles FCKH and AIOC are alike; and the Triangles CFH and AIC, are alſo ſimilar. And the Triangles BAI and BFH are likewiſe ſimilar.

From this Explanation, I ſay, the Square of the Area of the given Triangle will be $BFq \times IAq = BF \times BA \times CA \times CF$. In Words: The Square of BF (the half Sum of the Sides) multiply'd into the Square of IA ($= IE = IO$) will be equal to the ſaid half Sum multiply'd into all the three Differences.

For $IA : BA :: FH : BF$, and $IA : CF :: AC : FH$; becauſe the Triangles are ſimilar. By *Euclid, Lib.* 6, *Prop.* 4.

Wherefore multiplying the Extreams and Means in both, it will be $IAq \times BF \times FH = BA \times CA \times CF \times FH$; but FH being on both Sides of the Equation, it may be rejected; and then multiply each Part by BF, it will be $BFq \times IAq = BF \times BA \times CA \times CF$. Which was to be demonſtrated.

§ VI. *Of a* TRAPEZIUM.

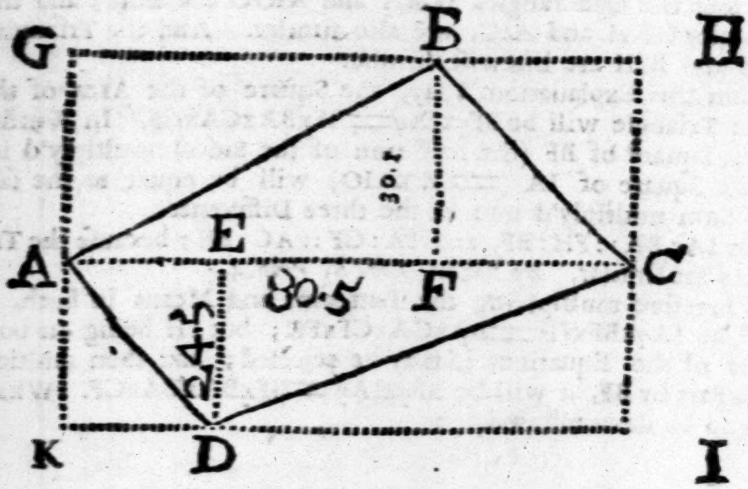
A Trapezium is a Figure having four unequal Sides and oblique Angles. To find the Area or ſuperficial Content thereof, this is

The *RULE*.

Add the two Perpendiculars together, and take half the Sum, and multiply that half Sum by the Diagonal, or multiply the whole Sum by half the Diagonal, the Product is the Area. Or you may find the Area's of the two Triangles, ABC and ACD, (by Section V.) and add thoſe two Area's together, the Sum ſhall be the Area of the Trapezium.

Or laſtly, Multiply the whole Diagonal by the Sum of the Perpendiculars, and take half the Product for the Area of the Trapezium.

$BF = 30.1$



BF=30.1
DE=24.5

Sum 54.6

Half 27.3
AC=80.5

1365
2184

Area 2197.65

Let ABCD be a Trapezium given, the Diagonal whereof is 80.5, and the Perpendicular BF. 30.1. and the Perpendicular DE 24.5, theſe two added together, the Sum is 54.6, the half thereof is 27.3, which multiply'd by the Diagonal 80.5 the Product is 2197.65, which is the Area of the Trapezium; or if 40.25, half the Diagonal, be multiply'd by 54.6, the whole Sum of the Perpendiculars, the Product is 2197.65, the ſame as before.

By

By Scale *and* Compaſſes.

Extend the Compaſſes from 2 to 54.63; that Extent will reach from 80.5 to 2197.65, the Area.

DEMONSTRATION.

This Figure A B C D, is compos'd of two Triangles; the Triangle A B C is half the Parallelogram A G H C; Alſo the Triangle ACD is equal to half the Parallelogram ACIK, as was prov'd Sect. V; wherefore the Trapezium ABCD is equal to half the Parallelogram GHIK. To find the Area HI = BF + DE, therefore $\frac{1}{2}$ HI×AC (= KI = GH) = Area of the Trapezium. Which was to be prov'd.

§ VII. *Of Irregular* FIGURES.

IRregular Figures are all ſuch as have more Sides than four, and the Sides and Angles unequal. All ſuch Figures may be divided into as many Triangles as there are Sides wanting two. To find the Area of ſuch Figures, they muſt be divided into Trapeziums and Triangles, by Lines drawn from one Angle to another, and ſo find the Area's of the Trapeziums and Triangles ſeverally, and then add all the Area's together, ſo will you have the Area of the whole Figure.

Let ABCDEFG be an irregular Figure given to be meaſur'd; firſt, draw the Lines AC and GD, and thereby divide the given Figure into two Trapeziums, ACGD and GDEF, and the Triangle ABC; of all which I find the Area's ſeverally.

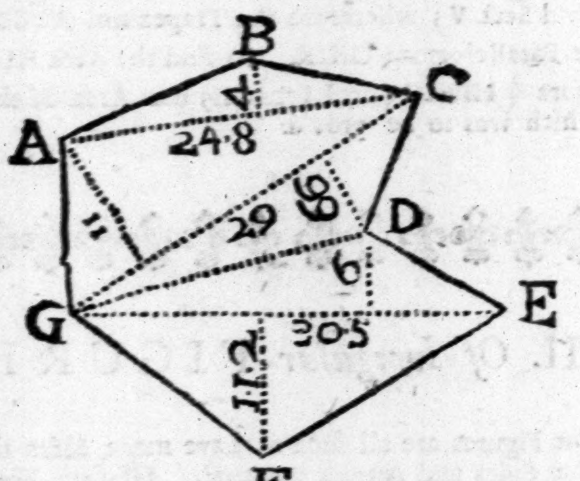
Firſt, I multiply the Baſe AC by half the Perpendicular, and the Product is 49.6, the Area of the Triangle ABC.

Then

Then, for the Trapezium ACGD, the two Perpendiculars, 11 and 6.6, added together, make 17.6; the half thereof is 8.8, multiply'd by 29, the Diagonal, the Product is 255.2, the Area of that Trapezium.

And for the Trapezium GDEF, the two Perpendiculars, 11.2 and 6, added together, make 17.2 the half thereof is 8.6, which multiply'd by 30.5, the Diagonal, the Product is 262.3, the Area thereof. All theſe Area's added together, make 567.1, and ſo much is the Area of the whole irregular Figure.

See the WORK.



24.8 Baſe AC
2 half Perpendicular

49.6 Area of ABC.

11
6.6 Perpendiculars.

17.6 Sum.

8.8 half.
29 Diag. CG.

792
176

255.2 Area of ACGD.

11.2

11.2	Perpendiculars.	30.5	
6.		8.6	
17.2	Sum	1830	
8.6	half Sum.	2440	
		262.30	Area of GDEF.
		255.2	Area of ACGD.
		49.6	Area of ABC.
		567.1	Sum of the Areas

This Figure being compos'd of Triangles and Trapeziums, and thoſe Figures being ſufficiently demonſtrated in the Vth and VIth Sections aforegoing, it will be needleſs to mention any thing of the Demonſtration thereof in this Place.

§ VIII. *Of Regular* POLYGONS.

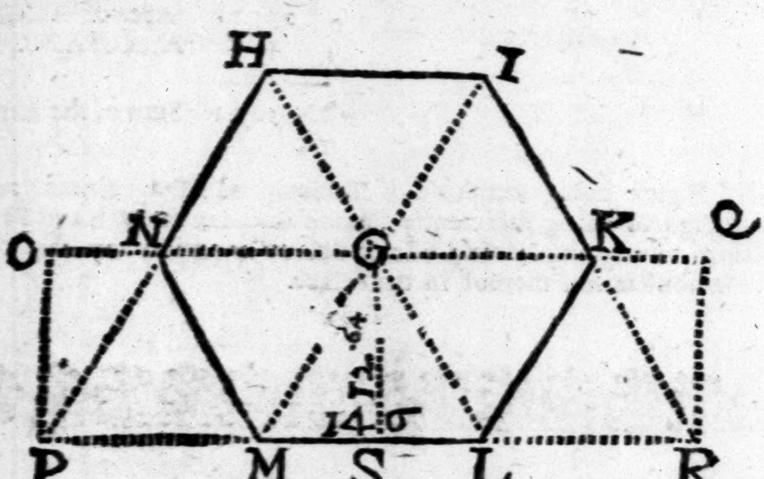
REgular Polygons are all ſuch Figures as have more than four Sides, all the Sides and Angles thereof being equal. Polygons are denominated from the Number of their Sides and Angles.

If the Figure conſiſts of		Equal Sides and Angles, it is call'd a regular	
	5		Pentagon.
	6		Hexagon.
	7		Heptagon.
	8		Octagon.
	9		Enneagon.
	10		Decagon.
	11		Endecagon.
	12		Dodecagon.

To find the Area or ſuperficial Content of any regular Polygon, this is

The RULE.

Multiply the whole Perimeter, or Sum of the Sides, by half the Perpendicular, let fall from the Center to the Middle of one of the Sides; or multiply the half Perimeter by the whole Perpendicular, and the Product is the Area.




```
14.6
   3
———
43.8 half Sum of the Sides.
———
12.64 the Perpendicular.
 43.8 half Sum.
———
10112
3792
5056
———
553.632 Area.
```

```
    14.6
       6
———
    87.6 Sum of the Sides.
    6.32 half Perpend.
———
   1752
  2628
 5256
———
553.632 Area.
```

Let

Let HIKLMN be a regular Hexagon, each Side thereof being 14.6, the Sum of all the Sides is 87.6, the half Sum thereof is 43.8, which multiply'd by the Perpendicular GS 12.64, the Product is 553.63: Or if 87.6, the whole Sum of the Sides be multiply'd by half the Perpendicular 6.32. the Product is 553.632, the ſame as before, which is the Area of the given Hexagon.

By Scale *and* Compaſſes.

Extend the Compaſſes from 1 to 12.6, that Extent will reach from 43.8, the ſame Way to 553.63: Or extend from 2 to 12.6, that Extent will reach from 87.6 to 553.63.

DEMONSTRATION.

Every regular Polygon is equal to the Parallelogram, or long Square, whoſe Length is equal to half the Sum of the Sides, and Breadth equal to the Perpendicular of the Polygon, as appears by the foregoing Figure; for the Hexagon HIKLMN is made up of ſix equilateral Triangles: And the Parallelogram OPQR is alſo compos'd of ſix equal and equilateral Triangles, that is, five whole ones, and two Halves; therefore the Parallelogram is equal to the Hexagon.

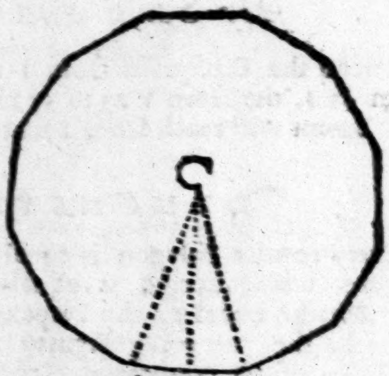
A TABLE *for the more ready finding the Area of a Polygon.*

Number of Sides.	Names.	Multipliers.
3	Trigon	.433013
4	Tetragon	1.000000
5	Pentagon	1.720477
6	Hexagon	2.598076
7	Heptagon	3.633959
8	Octagon	4.828427
9	Enneagon	6.181827
10	Decagon	7.694209
11	Endecagon	8.514250
12	Dodecagon	9.330125

Multiply the Square of the Side by the tabular Number, and the Product is the Area of the Polygon.

How to find theſe tabular Numbers.

Theſe Numbers are found by Trigonometry, thus: Find the Angle at the Center of the Polygon, by dividing 360 Degrees by the Number of Sides of the Polygon.



Example. Suppoſe each Side of the Dodecagon annex'd be 1, and the Area be requir'd.

Divide 360 by 12, (the Number of Sides) the Quotient is 30 Degrees for the Angle ACB; the half thereof is 15, the Angle DCB, whoſe Complement to 90 Degrees is 75 Degrees, the Angle CBD: Then ſay,

As s. DCB 15 Degrees,	C. Ar.	0.587004
To .5, the half Side DB, Log.		1.698970
So is s. CBD 75 Degrees,		9.984944
To the Perpendicular CD 1.866025		0.270918

Then 1.866025 multiply'd by .5, (the half Side) the Product is 9.330125, the Area of the Dodecagon requir'd.

§ IX. Of

§ IX. *Of a* CIRCLE.

A Circle is a plain Figure, contain'd under one Line, which is call'd a Circumference, unto which all Lines, drawn from a Point in the Middle of the Figure, call'd the Center, and falling upon the Circumference thereof, are all equal the one to the other. The Circle contains more Space than any plain Figure of equal Compass.

Problem 1. Having the Diameter and Circumference, to find the Area.

The RULE.

Every Circle is equal to a Parallelogram, whose Length is equal to half the Circumference, and the Breadth equal to half the Diameter; therefore multiply half the Circumference, by half the Diameter, and the Product is the Area of the Circle.

35.5 Half Circumference
11.3 Half Diameter.

1065
355
355

401.15 Area.

Thus, if the Diameter of a Circle (that is, the Line drawn cross the Circle through the Center) be 22.6, and if the Circumference be 71, the half of 71 is 35.5, and the half of 22.6 is 11.3; which multiply'd together, the Product is 401.15, which is the Area of the Circle.

DEMONSTRATION.

Every Circle may be conceiv'd to be a Polygon of an infinite Number of Sides, and the Semidiameter must be equal to the Perpendicular of such a Polygon, and the Circumference of the Circle equal to the Periphery of the Polygon; therefore half the Circumference, multiply'd by half the Diameter, gives the Area, as aforesaid.

Or, (with *F. Ignat. Gaston Pardies*) "Every Circle is equal to "a Rectangle-Triangle, one of whose Legs is the Radius, and "the other a right Line equal to the Circumference of the Cir- "cle: For such a Triangle will be greater than any Polygon in- "scrib'd, and less than any Polygon circumscrib'd, (by the 24th, "25th, 26th, and 27th Articles of the fourth Book of his Ele- "ments of Geometry) and therefore must be equal to the Circle.

"For (says he) should it be greater than the Circle, be the Ex- "cess as little as it will, a Polygon may be circumscrib'd, whose "Difference, from the Circle, shall be yet less than the Diffe- "rence between that Circle and the Rectangle-Triangle; and "that that Polygon will be less than the Triangle, is absurd; "and if it be said, that this rectangled Triangle is less than the "Circle, an inscrib'd Polygon may be made, which shall be "greater than that Triangle; which is impossible.

"This cannot but be admitted as a Principle, That if two de- "terminate Quantities, A and B, are such, that if every imagi- "nable Quantity, which is greater or less than A, is also grea- "ter or less than B, these two Quantities A and B must be "equal.

"And this Principle being granted, which is in a manner self- "evident, it may directly be prov'd, that the Triangle (before "mention'd) is equal to the Circle; because every imaginable "inscrib'd Figure, which is less than the Circle, is also less than "the Triangle; and every circumscrib'd Figure greater than "the Circle, is also greater than the Triangle.

Problem 2. Having the Diameter of a Circle, to find the Circumference.

A

As 7 to 22, ſo is the Diameter to the Circumference.
Or, as 113 to 355, ſo is the Diameter to the Circumference.
Or, as 1 to 3.141593, ſo is the Diameter to the Circumference.

Let the Diameter (as in the former Circle) be 22.6, this multiply'd by 22, and the Product is 497.2; which divided by 7, gives 71.028 for the Circumference. Or (by the ſecond Proportion) if 22.6 be multiply'd by 355, the Product will be 8023. this divided by 113, the Quotient is 71, the Circumference. Or, (by the third Proportion) if 22.6 be multiply'd into 3.141593, the Product is 71.0000018, the Circumference; which two laſt Proportions are the moſt exact.

```
   22.6              355
     22              22.6
  ------            ------
    452             2130
   452              710
 ---------         710
7)497.2(71.028     ------
               113)8023.0(71
                    791
                   -----
                    113
                    113
                   -----
                    ...

  3.141593
      22.6
 ----------
  18849558
  6283186
 6283186
 ----------
 71.0000018
```

By Scale *and* Compaſſes.

Extend the Compaſſes from 7 to 22, or from 113 to 355, or from 1 to 3.14159; that Extent will reach from 22.6 to 71.

The Proportion of the Diameter of a Circle, to the Circumference, was never yet exactly found, notwithſtanding many eminent learned Men have labour'd very far therein; amongſt which, the excellent *Van Culen* hath hitherto out-done all, in his having calculated the ſaid Proportion to 36 Places of Decimals, which are engraven upon his Tomb-ſtone in St. *Peter*'s Church in *Leyden*; which Numbers are theſe.

 Diameter,

Diameter.

1.00000.00000.00000.00000.00000.00000.00000

Circumference.

3.14159.26535.89793.23846.26433.83279.50288

Of which large Number theſe ſix Places, 3.14159, anſwering to the Diameter 1.00000, may be ſufficient; of the three Proportions, as 7 to 22. 113 to 355, and 1 to 3.14159, I ſhall leave my Reader to uſe which of them he pleaſes, but ſhall commend the laſt two as moſt exact, tho' the firſt be moſt in Uſe; but in the following Work I ſhall uſe ſometimes one of them, and ſometimes another, but for the moſt Part that of *Van Culen*, as being moſt exact.

Problem 3. Having the Circumference of a Circle, to find the Diameter.

As 1 is to .318309. ſo is the Circumference to the Diameter.
Or, as 355 to 113, ſo is the Circumference to the Diameter.
Or, as 22 to 7, ſo is the Circumference to the Diameter.

Let the Circumference be 71, (as in the former Circle) if .318309 be multiply'd by 71, (as by the firſt Proportion) the Product will be 22.599939 for the Diameter. Or, by the ſecond Proportion, 113 multiply'd by 71, the Product is 8023; which divided by 355, the Quotient will be 22.6, the Diameter. Or, by the third Proportion, 71 multiply'd by 7, the Product is 497; this divided by 22, the Quotient is 22.5909, the Diameter.

.318309	113	71
71	71	7
318309	113	22)497(22.59
2228163	791	44
22.599939	355)8023(22.6	57
	710	44
	923	130
	710	110
	2130	200
	2130	198
		2

Thus, by both the firft Proportions, the Diameter is 22.6, but by the laft it falls fomething fhort.

By Scale *and* Compaffes.

Extend the Compaffes from 3.14159 to 1, that Extent will reach from 71 to 22.6, which is the Diameter fought.

Or you may extend from 1 to .318309.

Or from 22 to 7.

Or from 355 to 113; the fame will reach from 71 to 22.6, as before,

NOTE, That if the Circumference be 1, the Diameter will be .318309.

Problem 4, Having the Diameter of a Circle, to find the Area.

All Circles are in Proportion one to another, as are the Squares of their Diameters, (by *Euc.* 12. 2.) Now the Area of a Circle, whofe Diameter is 1, will be .785398, according to *Van Culen*'s Proportion before mention'd; but for Practice .7854 will be fufficient: Therefore,

As 1 the Square of the Diameter 1) is to .7854, fo is 510.76 (the Square of 22.6, the Diameter of the given Circle) to 401.51, (the Area of the given Circle:) But,

Accor-

According to *Metius*'s Proportion;

As 452 : 355 :: 510.76 : 401.15, the ſame as before.

But, if you uſe *Archimedes*'s Proportion:

As 14 : 11 :: 510.76 : 401.31; which Area is greater than by the two former Proportions; though in ſmall Circles this is near enough the Truth.

See the Working of all theſe.

```
  22.6 Diameter of the former Circle.
  22.6
 -----
  1356
  452
 452
 -----
 510.76 The Square of the ſaid Diameter.

As 1 : .7854 :: 510.76
                .7854
              -------
               204304
              255380
             408608
            357532
            ------------
            401.150904 The Area.
```

By

By Scale *and* Compasses.

The Extent from 1 to 22.6, being twice turn'd over from .7854, will fall at the last upon 401.15, the Area.

```
113
  4
---
452

As 452 : 355 :: 510.76
                  355
               ------
               255380
              255380
             153228
             --------
452)181319.80(401.15
    1808
    -----
     519
     452
     ---
      678
      452
      ---
      2260
      2260
      ----
      ....

As 14 : 11 :: 510.76
                  11
              ------
14(561836(401.31
   56
   --
     18
     14
     --
      43
      42
      --
       16
       14
       --
        2
```

Problem

Problem 5. Having the Circumference of a Circle, to find the Area.

Becauſe the Diameters of Circles are proportional to the Circumferences; that is, as the Diameter of one Circle is to its Circumference; ſo is the Diameter of another Circle to its Circumference. Therefore the Area's of Circles are to one another, as the Squares of the Circumferences. And if the Circumference of a Circle be 1, the Area of that Circle will be .07958; then the Square of 1 is 1, and the Square of 71 (the Circumference of the former Circle) is 5041. Therefore it will be,

```
         Sq. Cir.  Area   Sq. Circumf.
         As  1 : .07958 :: 5041
                   5041
                ----------
                   .7958
22                31832
 4               397900
--              ----------
88               401.16278 Area.
```

Or thus:

```
        As 88 : 7 :: 5041
                        7
               -----------
        88|35287|400.98 Area.
           352
        -------
              870
              792
             -----
355           780
  4           704
-----        -----
1424           76
```

Area.

Or, As 1420 : 113 :: 5041 : 401.15.

Problem 6. By having the Diameter, to find the Side of a Square that is equal in Area to that Circle.

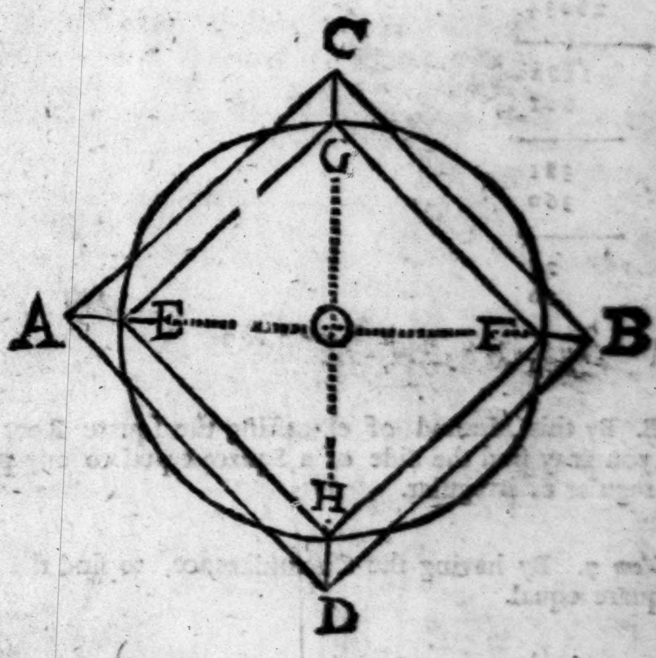
If

If the Diameter of a Circle be 1, the Side of a Square equal thereunto will be .8862. Therefore,

As 1 : .8862 : : 22.6 (the Diameter.)
22.6

53172
17724
17724

To 20.02812 the Side of the Square AC.



Let the Diameter of the Circle EF or GH, be 22.6, (as before) to find the Side of the Square AC, AD, &c. If .8862 be multiply'd by 22.6, the Product is 20.02812, which is the Side of a Square, equal in Area to the Circle given; for if 20.02812 be multiply'd square-wise, that is, by it self, it will produce 401.1255907344, which is nearly equal to the Area found in the last Problem.

You

You may find the Side of the Square equal, by extracting the ſquare Root out of the Area of the given Circle.

```
 401.15(20.0287295 Side of the Square.
 4
 ----------
4002(01.1500
       8004
 ----------
40048(349600
      320384
      ------
       29216
       28034
      ------
        1182
         801
      ------
         381
         360
      ------
          21
          20
      ------
           1
```

N. B. By this Method of extracting the ſquare Root of the Area, you may find the Side of a Square equal to any plain Figure, regular or irregular.

Problem 7. By having the Circumference, to find the Side of the Square equal.

If the Circumference of a Circle be 1, the Side of the Square equal will be .2821. Therefore,

```
As 1 : .2821 :: 71 (the Circumference.)
           71
      -------
         2821
        19747
      -------
      20.0291 the Side of the Square.
```

Problem

Problem 8. Having the Diameter, to find the Side of a Square which may be inſcrib'd in that Circle.

If the Diameter of a Circle be 1, the Side of the Square inſcrib'd will be .7071. Therefore,

As 1 : .7071 :: 22.6
22.6

42426
14142
14142

To 15.98046 the Side EG inſcrib'd.

Or, if you ſquare the Semidiameter, and double that Square, the ſquare Root of the doubled Square will be the Side of the Square inſcrib'd. For (by *Euclid* 1.47.) the Square of the Hypothenuſe E G is equal to the Sum of the other two Legs, EO and OG.

11.3 Semidiameter.
11.3

339
113
113

127.69 the Squ. of EO, which double, becauſe EO = OG.
2

255.38(15.98 Root which is the Side of the Square.
1

25)155
125

309)3038
2781

3188)25700
25504

6

Problem

Problem 9. Having the Circumference, to find the Side of a Square which may be inſcrib'd.

If the Circumference be 1, the Side of the Square inſcrib'd will be .2251. Therefore,

As 1 : .2251 : : 71
71

2251
15757

15.9821 the Side of the Square EQ.

Becauſe that in each of the four laſt Problems, *viz.* the 6th, 7th, 8th, and 9th, there is a Proportion laid down, it will be eaſy to work them with Scale and Compaſſes; for if you extend the Compaſſes from the firſt to the ſecond, that Extent will reach from the third to the fourth. As in the laſt Problem, where the Proportion is as 1 to .2251, ſo is 71 to the Side of the Square 15.9821. Here extend the Compaſſes from 1 to .2251; that Extent will reach from 71 to 15.98; and ſo of the reſt. But the 5th muſt be wrought like the 4th, thus; extend the Compaſſes from 1 to 71, that Extent, turn'd over the ſame Way from .07958, will fall, at the laſt, upon 401.15,

Problem 10. Having the Area, to find the Diameter.

If the Area of a Circle be 1, the Square of the Diameter thereof is 1.2732. Therefore,

As

Area. Sq. Diam. Area
As 1 : 1.2732 :: 401,15
401.15

63660
12732
12732
50928

510.744180

510.744180(22.599 the Diam.
4

42)110
84

445)2674
2225

4509)44941
40581

45189)436080
406701

29379

By Scale *and* Compasses.

Extend the Compasses from 1 to 1.27323; that Extent will reach from 401.15 to 510.74, *&c.* Then divide the Space between 1 and 510.74 into two equal Parts, and you'll find the middle Point at 22.6. Or you may divide the Space upon the Line of Numbers, between 401.15 and .7854, into two equal Parts, and one of those Parts will reach from 1 to 22.6, the Diameter sought.

Problem 11. Having the Area, to find the Circumference.

If the Area of a Circle be 1, the Square of the Circumference will be 12.56637. Therefore,

Ar. Sq. Circumf. Area.
As 1 : 12.56637 :: 401.15
401.15

6283185
1256637
1256637
50265480

5040.99932550

Circumf.
5040.99932550(70.9999 Root
49

1409)14099
12681

14189)141893
127701

141989)1419225
1277901

1419989)14132450
12779901

1352549

By

By Scale *and* Compaſſes.

Divide the Space between 401.15 and .07958, upon the Line into two equal Parts; one of thoſe Parts will reach from 1 to 71, the Circumference ſought.

Problem 12. Having the Area, to find the Side of a Square inſcrib'd.

If the Area of a Circle be 1, the Area of a Square inſcrib'd within that Circle will be .6366. Therefore,

As 1 : 401.15 :: .6366
.6366

240690
240690
120345
240690

255.372090 (15.98 Root, which is the Side of the Sq. ſought.
1

25)155
125

309)3037
2781

3188)25629
25504

11690

The ſame Reaſon may be given for the laſt Proportion that was given before for the Proportion of Circles to the Squares of their Diameters and Circumferences; for not only the Squares of the Diameters and Circumferences are in Proportion to the Circles they belong to, but alſo all Figures inſcrib'd or circumſcrib'd, have the Squares of their like Sides proportional to the Circles they are inſcrib'd in, or circumſcrib'd about; and alſo to the Figures themſelves; The Square of any Side of one Figure is to the Area of that Figure, as the Square of the like Side of another ſimilar Figure is to the Area thereof, as you may find prov'd at large in *Euclid*, *Sturmius's*, *Matheſis Enucleata*, and other Authors, but will be too large to inſert in this Place.

By Scale *and* Compaſſes.

Extend the Compaſſes from 1 to 401.15, that Extent will reach from .6366 to 255.37; the half Space between that and 1 is at 15.98, the Side of the Square.

Prob*lem*

Problem 13. Having the Side of a Square, to find the Diameter of the circumscribing Circle.

If the Side of a Square be 1, the Diameter of a Circle that will circumscribe that Square, will be 1.4142. Therefore,

```
As 1 : 1.4142 : : 15.98
       15.98
      ------
      113136
     127278
     70710
    14142
   --------
  22.598916 the Diameter sought.
```

By Scale *and* Compasses.

Extend the Compasses from 1 to 1.4142, and that Extent will reach from 15.98 to 22.6, the Diameter sought.

Problem 14. Having the Side of a Square, to find the Diameter of a Circle equal.

If the Side of a Square be 1, the Diameter of a Circle equal thereunto will be 1.128. Therefore,

```
Side   Diam.   Side of a Square.
As 1 : 1.128 : : 20.0291
                   1.128
               ---------
                1602328
               400582
              200291
             200291
           -----------
           22.5928248 Diam.
```

By Scale *and* Compasses.

Extend the Compasses from 1 to 1.128; that Extent will reach from 20.0291 (the Side of the Square given) to 22.6, the Diameter of a Circle sought.

Problem 15. Having the Side of a Square, to find the Circumference of the circumfcribing Circle.

If the Side of a Square be 1, the Circumference of a Circle that will encompafs that Square, will be 4.443. Therefore,

Side Sq. Circum. Side Sq.
As 1 : 4.443 : : 15.98
15.98

35544
39987
22215
4443

70.99914 the Circumf.

By Scale *and* Compaffes.

Extend the Compaffes from 1 to 4.443, that Extent will reach from 15.98 to 71, the Circumference.

Problem 16. Having the Side of a Square, to find the Circumference of a Circle that will be equal thereunto.

If the Side of the Square be 1, the Circumference of a Circle that will be equal thereunto, fhall be 3.545, Then,

As 1 : 3.545 : : 20.0291
3.545

1001455
801164
1001455
600873

71.0031595 the Circum.

By Scale *and* Compaffes.

Extend the Compaffes from 1 to 3.545, that Extent will reach from 20.0291 to 71, the Circumference fought.

In feveral of the foregoing Problems, where the Diameter and Circumference is requir'd, the Anfwers are not exactly the fame as the Diameter and Circumference of the given Circle, but are fome

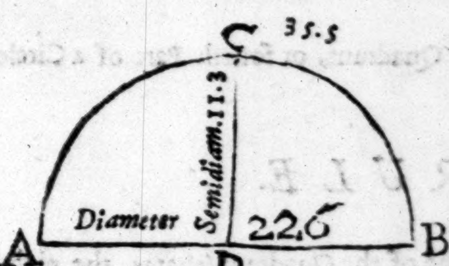
sometimes too little. as in the two last Problems, where the Answers in each should be 71, the one being too much, and the other too little. The Reason of this is, the small Defect that happens to be in the Decimal Fractions, they being sometimes too great, and sometimes too little; yet the Defect is so small, that it is needless to calculate them to more Exactness.

§ X. *Of a* SEMICIRCLE.

TO find the Area of a Semicircle, this is

The *RULE.*

Multiply the fourth Part of the Circumference of the whole Circle (that is, half the Arch-Line) by the Semidiameter, the Product is the Area.



Let ABC be a Semi-circle whose Diameter 22.6, and the half Circumference, or Arch-line, ACB, is 35.5; the half thereof is 17.75, which multiply by the Semidiameter 11.3, and the Product is 200.575, the Area of the Semicircle.

17.75 the half Arch-line.
11.3 the Semidiameter.

5325
1775
1775

200.575 The Area of the Semicircle.

By Scale and Compasses.

Extend the Compasses from 1 to 11.3; that Extent will reach from 17.75 to 200.575, the Area.

If only the Diameter of the Semicircle be given, you may say, by the *Rule of Three*.

As 1 is to .3927, so is the Square of the Diameter to the Area.

By Scale and Compasses.

Extend the Compasses from 1 to the Diameter 22.6; that Extent turn'd twice from .3927, will reach at the last, to 200.575.

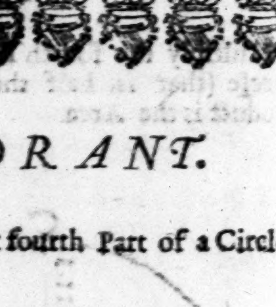
§ XI. *Of a QUADRANT.*

TO find the Area of a Quadrant, or fourth Part of a Circle, this is

The RULE.

Multiply half the Arch-line of the Quadrant (that is, the eighth Part of the Circumference of the whole Circle) by the Semidiameter, and the Product is the Area of the Quadrant.

Let ABC be a Quadrant, or fourth Part of a Circle, whose Radius, or Semidiameter, is 11.3, and the half Arch-line 8.875; these multiply'd together, the Product is 100.2875 for the Area.

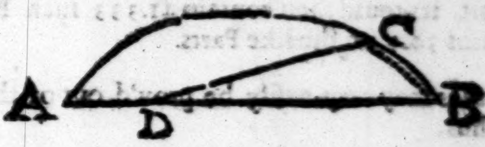


These are the Rules and Ways commonly given for finding the Area of a Semicircle and Quadrant; but, I think, it is as good a Way, to find the Area of the whole Circle, and then take half that Area for the Semicircle, and fourth Part for the Quadrant.

Before I proceed to shew how to find the Area of the Sector, and Segment of a Circle, I shall shew how to find the Length of the Arch-line, both *Geometrically* and *Arithmetically*.

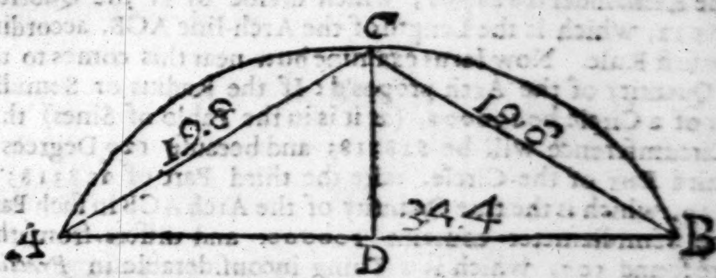
To find the Length of the Arch-line Geometrically.

Divide the Chord-line AB into four equal Parts, and set one of these Parts from B to C, and draw a Line from C to three of those Parts at D; so shall CD be equal to half the Arch-line ACB.



To find the Length of the Arch-line Arithmetically.

Multiply the Chord of half the Segment AC or CB by 8, and from the Product subtract the Chord of the whole Segment AB, and divide the Remainder by 3, the Quotient is the Arch-line ACB sought.



19.8 AC
8

158.4
34.4 AB

3)124

Arch-line 41.333

Arch-line

Another Way.

From the double Chord of half the Segment's Arch, subtract the Chord of the Segment, one third Part of the Difference added to the double Chord of half the Segment's Arch, the Sum is the Arch-line of the whole Segment.

Thus, if AC 19.8 be doubled, it makes 39.6; from which if you subtract 34.4, the Remainder is 5.2, which divided by 3, the Quotient is 1.733; this added to 39.6, (the double Chord of the half Segment) the Sum is 41.333. So if the Arch-line ACB was stretch'd out strait, it would then contain 41.333 such Parts as the Chord AB contains 34.4 of the like Parts.

These two Rules may very easily be prov'd out of the Table of natural Sines; thus,

Suppose (in the former Figure) the Arch ACB to contain 120 Degrees, the natural Sine of half, *viz.* of 60 Degrees, is 86602, which being doubled, is 173204, which is the Chord of the whole 120 Deg. that is AB. Then, to find the Chord of the half Arch, *viz.* AC 60 Degrees, the half of it 30 Degrees, the natural Sine thereof is 50000; which, doubled, makes 100000 for the Chord AC; then, according to the first Rule, multiply 100000 by 8, the Product is 800000; from which subtract 173204, (the Chord AB) and the Remainder is 626796; which divide by 3, the Quotient is 208932, which is the Length of the Arch-line ACB, according to the first Rule. Now let us examine how near this comes to the true Quantity of the Arch propos'd: If the Radius or Semidiameter of a Circle be 100000, (as it is in the Table of Sines) then the Circumference will be 628318; and because 120 Degrees is the third Part of the Circle, take the third Part of 628318; is 209439, which is the true Quantity of the Arch ACB in such Parts as the Semidiameter contains 100000. and differs from that before found 507, which is a Thing inconsiderable in *Practical Mensuration.* The latter of the foregoing Rules agrees exactly with the former, and therefore the Difference will be the same as above; either of the Rules gives the Quantity of the Arch-line too little, and the greater the Arch, the greater the Error. If you know the Degrees that are contain'd in the Segment's Arch, and would have the Arch-line very exactly, you may reason thus by the *Rule of Three.*

As the Circle's Periphery in Degrees : is to its Periphery in equal Parts : : so is the Arch in Degrees and decimal Parts : to the same Arch in equal Parts.

Sup-

Suppose the Circumference of a Circle be 71, and suppose the Arch to contain 52 Degrees, 15 Minutes, (the Decimal of 15 Minutes is .25) then say,

Deg.	Parts.	Deg.
As 360 :	71 ::	52.25

$$
\begin{array}{r}
52.25 \\
71 \\
\hline
5225 \\
36575 \\
\hline
36{,}0)370|9.75(10.305\ \textit{fere.} \\
36 \\
\hline
109 \\
108 \\
\hline
175
\end{array}
$$

So the 52 Degrees 15 Minutes will contain 10.305 of such Parts as the Circumference contains 71.

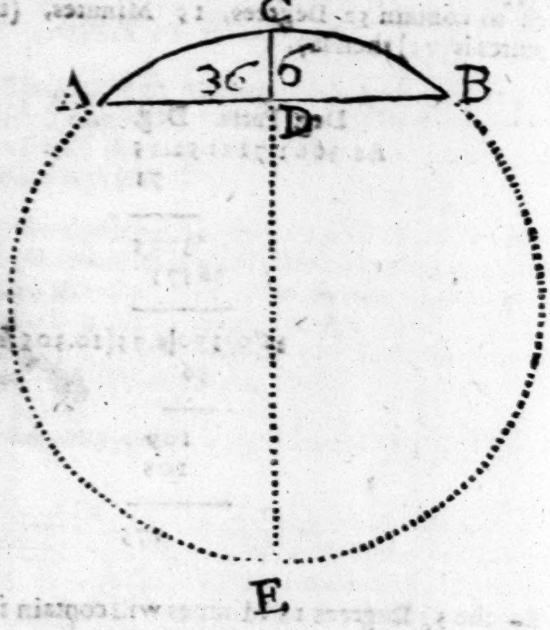
Thus have I shew'd several Ways of finding the Measure of the Curve-line of any Part of a Circle very near the Truth. The next Thing I shall shew, is,

How to find the Diameter of a Circle, by having the Chord and versed Sine of the Segment Arithmetically.

Because the Chord A B cuts the Diameter E C at right Angles, therefore the Semichord A D or D B is a mean proportional Line between the Parts of the Diameter C D and D E, (by *Euc.* 6. 13.) Therefore, if you square the Semichord A D or D B, and divide that Square by the versed Sine, C D, the Quotient will be the Part of the Diameter wanting; to which add the given versed Sine G D, and the Sum is the Diameter sought.

Example

Example. Let ACB be a Segment given, whose Chord AB is 36, and the versed Sine CD 6; half 36 is 18, which squar'd, makes 324, this divided by 6, the Quotient is 54, to which add 6, the Sum is 60, the Diameter of the Circle CE.



18 half the Chord.
18

144
18

6)324 the Squa. of AD

54 the Part wanting DE.
6 the versed Sine CD add.

60 the Diameter CE.

§ XII. *Of a Sector of a* CIRCLE.

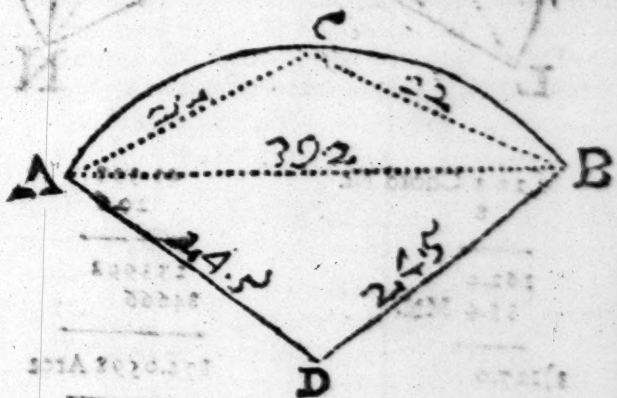
A Sector of a Circle is comprehended under two Radii, or Semidiameters, which are suppos'd not to make one right Line, and a Part of the Circumference; whence a Sector may

may be either lefs or greater than a Semicircle. To find the Area or fuperficial Content thereof, this is

The RULE.

Multiply half the Arch-line by the Semidiameter, and the Product is the Area.

Let ADBC be the Sector of a Circle given, whofe Semidiameter AD or BD is 24.5. and the Arch-line A C B (by the firft Rule, *pag* 107.) I find to be 45.6, the Half thereof, 22.8, being multiply'd by 24.5, (the Semidiameter) the Product is 558.6, which is the Area of the Sector ACBD.

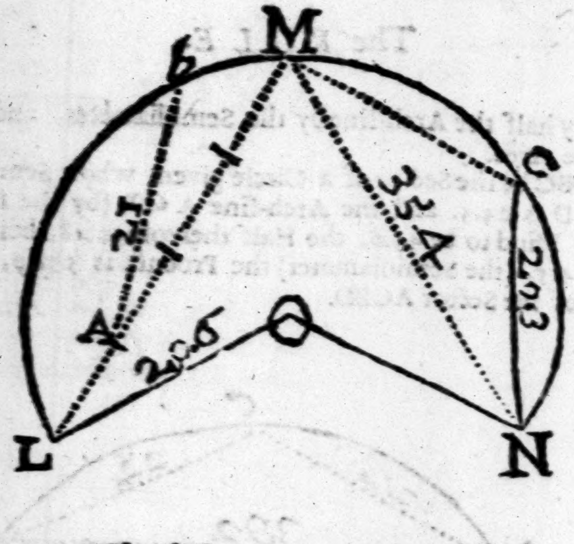


22	22.8 half Arch-line.
8	24.5 Semidiameter.
176	1140
39.2 Subtrahend.	912
	456
3)136.8	
45.6 Arch-line.	558.60 the Area.

Again. Let LMNO be a Sector greater than a Semicircle, whofe Semidiameter LO or NO is 20.6, and Line *b a* equal to a fourth Part of the Arch-line L*b*MN 21, the Double whereof is 42, equal to the Arch-line L*b*M or M*c*N; or by the Arithmetical Rule. *pag.* 107. the faid Arch is found to be 42.333; which multiply'd by 20.6, the Semidiameter makes 872.0598 for the Area of the Sector LMNO.

20.3

See the following WORK.

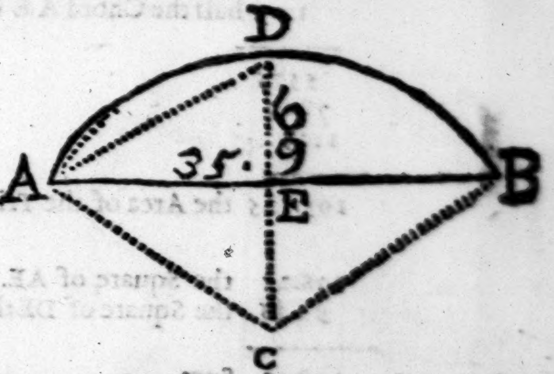


20.3 Chord Nc	42.333
8	20.6
162.4	253998
35.4 MN	84666
3)127.0	872.0598 Area
42.333 Arch-line	

§ XIII. *Of the Segment of a* CIRCLE.

A Segment of a Circle, is a Part terminated by a right-line less than the Diameter, call'd a Chord, and by a Part of the Circumference.

To find the Area of the Segment of a Circle, you must, first, find the Center of the whole Circle, and draw the two Semidiameters, thereby completing the Sector, as in the following Figure. Then (by the last Section) find the Area of the whole Sector CADBC, and then (by Sect. 5.) find the Area of the Triangle ABC, and subtract the Area of the Triangle out of the Area of the Sector, the Remainder is the Area of the Segment. Other-



Otherwise, you may, without describing the Figure, find the Semidiameter of the Circle by the Arithmetical Rule, *p.* 110 and by the Arithmetical Rule, *p.* 107, find the Arch-line; then multiply half the Arch-line by the Semidiameter, so have you the Area of the Sector. Then subtract the versed Sine from the Semidiameter, the Remainder is the Perpendicular of the Triangle; and multiply the half Chord by the Perpendicular, the Product is the Area of the Triangle. Then subtract the Area of the Triangle from the Area of the Sector, and the Remainder is the Area of the Segment.

See the WORK.

2)35 = AB

17.5
17.5

875
1225
175

9.6)306.25 (31.9
182   9.6 add.
865   41.5 the Diameter of the Circle.

1   20.75 the Semidiameter.
9.6 DE Subtrahend.

11.15 remains the Perpendicular E C.

11.15

11.15 the Perpendicular E C.
17.5 half the Chord A E or E B.

5575
7805
1115

195.125 the Area of the Triangle.

306.25 the Square of AE.
92.16 the Square of DE the versed Sine.

398.41 Sum.

The square Root thereof is 19.96 the Chord A D.

159.68
Sub. 35 the Chord AB.

WORK

3)124.68

2)41.56 the Arch-line.

20.78 half.
20.75 Semidiameter.

10390
14546
4156

From 431.1850 Area of the Sector.

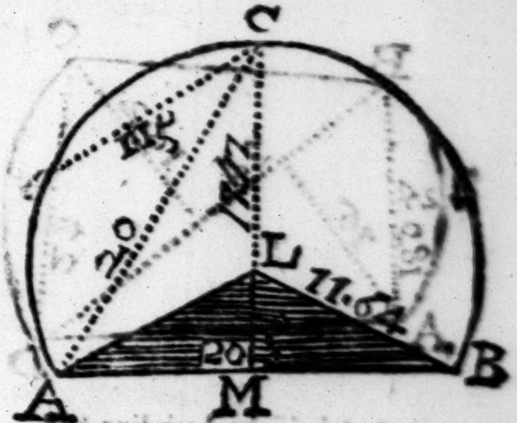
Subtract 195.125 Area of the Triangle.

Remains 236.060 Area of the Segment.

Again, Let M A C B M be a Segment greater than a Semicircle; observe the former Rules in all Respects, as in the last Example, only instead of subtracting the Area of the Triangle out of the Area of the Sector, here you must add it thereunto, as may plainly appear by the following Figure.

11.5

11.5
8
―――
92.0
20
―――
3)72
―――
24 half Arch line



11.64 Semidiam.
24
―――
4656
2328
―――
279.36 Area of the Sector LACBL.

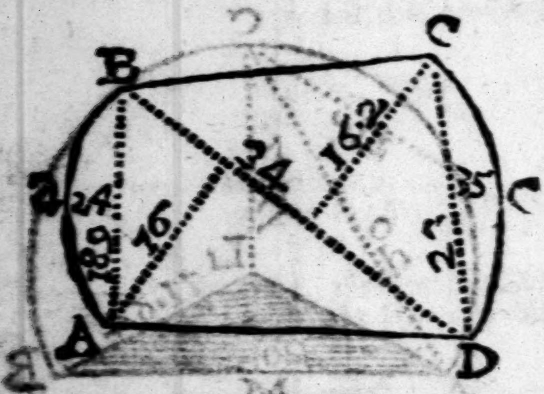
1317
11,64
―――
5.53 DM.

10.25 half the Base MA.
5.53 the Perpendicular LM.
―――
3075
5125
5125
―――
56.6825 the Area of the Triangle ALM
279.36 the Area of the Sector add.
―――
336.0425 the Area of the Segment sought

§ XIV. *Of Compound* FIGURES.

MIx'd or compound Figures, are such as are compos'd of rectilineal and curvilineal Figures together.

To find the Area of such mix'd Figures, you must find the Area of the several Figures of which the whole compound Figure is compos'd, and add all the Area's together, and the Sum will be the Area of the whole compound Figure.



16.2 } add
16.

32.2 Sum.

16.1 half.
34 Diagonal.

644
483

547.4 Area of the (Trapezium.

10.236 half the Arch-line AaB.
14.83 Semidiameter of the Arch AaB.

30708
81888
40944
10236

151.79988 Area of the Sector.

From 14.83 Semidiameter,
Subtract 3.4 versed Sine.

Rem. 11.43 Perpend. of the Triangle.
9.45 half the Chord AB.

5715
4572
10287

108.0135 the Area of the Triangle subtracted
151.7999 from the Area of the Sector.

43.7864 the Area of the Segment AaBA.

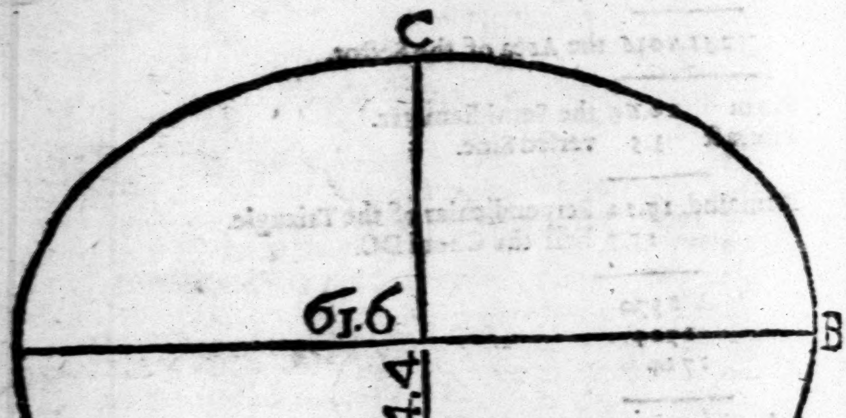
	12.19	half the Arch-line CcD.
	20.64	Semidiameter.
	4876	
	7314	
	2438	
	251.6016	the Area of the Sector.
From	20.64	the Semidiameter.
subtract	3.5	versed Sine.
Remaind.	17.14	Perpendicular of the Triangle.
	11.5	half the Chord DC.
	8570	
	1714	
	1714	
Subtr.	197.110	Area of the Triangle,
From	251.602	the Area of the Sector.
Rem.	54.492	the Area of the Segment CcDC.
	43.786	the Area of the Segment AaBA.
	547.4	the Area of the Trapezium.
Sum	645.678	the Area of the Whole.

§ XV. Of an ELLIPSIS.

AN Ellipsis, or Oval, is a Figure bounded by a regular Curve-line, returning into it self; but of its two Diameters, cutting each other in the Center, one is longer than the other, in which it differs from the Circle. To find the Area thereof, this is

The RULE.

Multiply the transverse Diameter by the Conjugate, and multiply that Product by .7854, this last Product is the Area of the Oval.



61.6 the transverse Diameter.
44.4 the conjugate Diameter.

2464
2464
2464

2735.04 the Rectangle.
.7854 the Area of Unity.

1094016
1367520
2188032
1914528

2148.100416 the Area of the Oval.

Demonstration. If you circumscribe any Ellipsis with a Circle, and suppose an infinite Number of Chord-lines drawn therein, all parallel to the conjugate Diameter, as those in the following Figure, then it will be,

As DA, the Diameter of the Circle, : is to Nn, the conjugate Diameter of the Ellipsis : : so is BaB, any Chord in the Circle, to bab, its respective Ordinate in the Ellipsis.

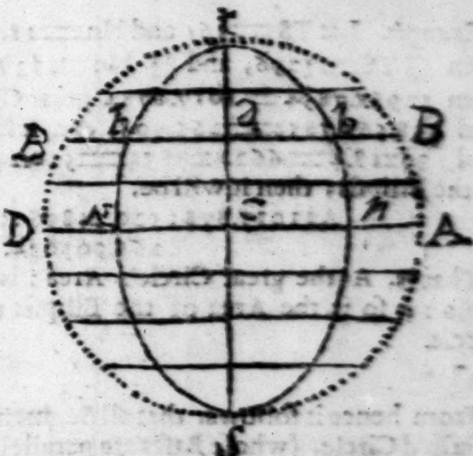
For,

For, according to to the Property of the Circle.

it is	1	$aS \times Ta = \square\, Ba$.
And		by the Property of the Ellipsis,
it is	2	$\square\, TC : NC :: aS \times Ta : \square\, ba$
1, 2.	3	$\square\, TC : \square\, NC :: \square\, Ba : \square\, ba$.
3, hence	4	$TC : NC :: Ba : ba$.
Conseq.	5	$2TC : 2NC : 2Ba : 2ba$.
That is	6	$DA : Nn :: BaB : bab$.

But the Sum of an infinite Series of such Chords, as Bab, do constitute the Area of the Circle. And the Sum of the like Series of their respective Ordinates as b a b, do constitute the Area of the Ellipsis.

Therefore, As TS : to Nn : : Circles Area : to the Ellipsis Area. But TS : Nn : : $\square$ TS : TS$\times$ Nn ; whence it follows that,



As $\square$ TS : Circles Area : : so is The Nn : Ellipsis Area.

Consequently, As 1 : to .7854 : : so is the Rectangle, or Product of the transverse and conjugate Diameter of any Ellipsis : to its Area.

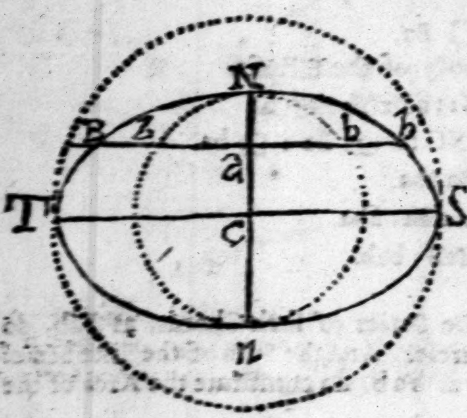
Hence it is easy to conceive, that the square Root of the Product of the transverse and conjugate Diameters, will be the Diameter of a Circle equal to the Ellipsis.

Hence also all Segments of an Ellipsis, and its circumscribing Circle, (whose Bases are parallel to the conjugate Diameter, and of the same Height) are in Proportion one to another, as their Bases are, that is,

As BaB : b a b : : Area Segment BTB : Area Segment bTb.

Or, As TS : Nn : Area Segment BTB : Area Segment bTb.

The Area of every Ellipsis is a mean Proportional between the Area's of its circumscribing and inscrib'd Circles.



The Truth of this may be easily deduc'd from the last; for its already prov'd, that $\square$ TS : TS x Nn : : circumscribing Circle's Area: Ellipsis Area.

But $\square$ TS : TS x Nn : : TS x Nn: $\square$ Nn. Therefore Ellipsis Area: inscrib'd Circles Area : : TS x Nn : $\square$ Nn.

Example. Let TS = 36, and Nn = 18.4.
Then $\square$ TS = 1296, and $\square$ Nn = 338.56.
Then 1296 x .7854 = 1017.8784 great Circles Area:
And 338.56 x .7854 = 265.905, *&c.* lesser Circles Area;
And 36 x 18.4 = 662.4 x .7854 = 520.24896, which is the Area of the Ellipsis; then it will be,

As 1017.878 : 520.24896 : : 520.24896 : 265.905024.

That is, As the great Circle's Area : is to the Area of the Ellipsis : : so is the Area of the Ellipsis to the Area of the lesser Circle.

From hence it follows, that all Segments of an Ellipsis, and its inscrib'd Circle, (whose Bases are parallel to the transverse Diameter, and have the same Height) are in Proportion one to another, as the Area of the Ellipsis and Circle are.

That is, As the Area of the Circle : to the Area of the Ellipsis : : so is the Segment bNh: to the Segment BNB,

Or, Nn : TS : : Area Segment bNb : Area Segment BNB.

§ XVI. *Of a* PARABOLA.

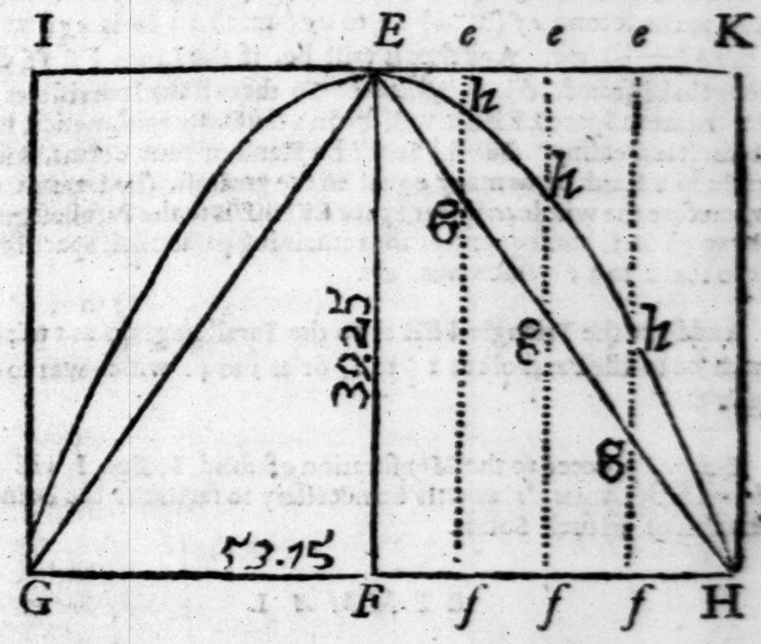
A PARABOLA is a curvilinear Figure, made by the Section of a Cone, being cut by a Plane parallel to one of its Sides.

Every

Every Parabola is two Thirds of its circumscribing Parallelogram; therefore to find the Area thereof, this is.

The RULE.

Multiply the Base, or greatest Ordinate, by the perpendicular Height, and multiply that Product by 2, and divide the last Product by 3, the Quotient will be the Area of the Parabola.



53.75 the Ordinate GH.
39.25 the Perpendicular EF.

26875
10750
48375
16125

2109.6875
2

3)4219.3750

1406.4583 the Area.

Demonstration. Let FH, the Semi-ordinate, be divided into four equal Parts, or into 8.16, *&c.* and through the Divisions draw Lines, as *e f*, *e f*, *&c.* parallel to the Axis EF. Suppose also EF to be 4.

Then, I say, the Parabolick Space EhHF is to the Parallelogram EKFH as 2 to 3; but to the Triangle EFH as 4 to 3.

 For

For, first, *gf*, *gf*, *gf*, &c are in continual arithmetical Proportion from the Nature of plain Triangles.

Secondly, $fe : ge :: ge : he$; but he in the Axis EF $= 0$, and in the first Parallel ef must be equal to $\frac{1}{4}$, in the next ef must be equal to $\frac{4}{4}$, in the third to $\frac{9}{4}$, and so on, in a duplicate arithmetical Progression.

For as $ef, (= 4) : ge \ (= 1) ::$ so is $ge \ (= 1)$: to $eh \ (= \frac{1}{4}.)$ And as the second $ef \ (= 4) ::$ to $eg \ (= 2) ::$ so is $eg \ (= 2)$: to $eh \ (= \frac{4}{4})$ &c. And thus it will be, if the Lines *Ff*, *ff*, &c. be again bissected, &c. *ad infinitum*. So that all the Indivisibles of the trilinear Space EKHhE will be in a duplicate arithmetical Progression increasing. But the Sum of a Rank of such Terms, is subtriple to a Rank of as many equal to the greatest, (by LEMMA 3); wherefore the whole trilinear Space EKHhE is to the Parallelogram as 1 to 3; and, consequently, the remaining parabolick Space must be to it as 2 to 3; which was, &c.

And since the Triangle FEH is to the Parallelogram as 1 to 2, it must be to the Parabola as $1\frac{1}{2}$ to 2, or as 3 to 4: which was to be prov'd.

Before I proceed to the Mensuration of solid Bodies, I will lay down such LEMMA's as will be necessary to facilitate the Demonstration of all such Solids.

LEMMA I.

In any Series of equal Numbers, (representing Lines or other Quantities) as 1, 1. 1, 1, &c. or 2, 2, 2, 2, &c. or 3, 3, 3, 3, &c. if one of the Terms be multiply'd into the Number of Terms, the Product will be the Sum of all the Terms in the Series.

LEMMA II.

If a Series of Numbers, in arithmetical Progressions, begin with a Cypher, and the common Difference be 1, as 0, 1, 2, 3, &c. (representing a Series of Lines or Roots beginning with a Point) if the last Term be multiply'd into the Number of Terms, the Product will be double the Sum of all the Series.

That

That is putting L $=$ the last Term, N $=$ the Number of Terms, and S $=$ the Sum of all the Series. Then will NL $= 2$S; consequently, $\frac{1}{2}$ NL $=$ S, *viz.* one half of so many times the greatest Term as there are Number of Terms in the Series.

Thus $\begin{cases} \dfrac{0+1+2+3+4}{4+4+4+4+4} = \dfrac{10}{20} & \begin{array}{l} \text{the Sum} = \frac{1}{2}\,NL. \\ = NL. \end{array} \end{cases}$

LEMMA III.

If a Series of Squares, whose Sides or Roots are in arithmetical Progression, beginning with a Cypher, *&c.* be infinitely continu'd; the last Term, being multiply'd into the Number of Terms, will be triple to the Sum of all the Series, *viz.* NLL $= 3$S; or, $\frac{1}{3}$ NLL $=$ S.

That is, the Sum of such a Series will be one third of the last, or greatest Term, so many Times repeated as there are Numbers of Terms in their Series.

Instances in square Numbers.

$$1\ \left\{ \frac{0+1+4}{4+4+4} = \frac{5}{12} = \frac{1}{3} + \frac{1}{12}. \right.$$

$$2\ \left\{ \frac{0+1+4+9}{9+9+9+9} = \frac{14}{36} = \frac{7}{18} = \frac{1}{3} + \frac{1}{18}. \right.$$

$$3\ \left\{ \frac{0+1+4+9+16}{16+16+16+16+16} = \frac{30}{80} = \frac{3}{8} = \frac{9}{24} = \frac{1}{3} + \frac{1}{24},\ \&c. \right.$$

From these Instances it is evident, that as the Number of Terms in the Series do increase, the Fraction, or excess above one third does decrease, the said Excess always being $\frac{1}{6N-6}$ which, if we suppose the Series to be infinitely continu'd, will quite vanish, and become nothing at all.

LEMMA

LEMMA IV.

If a Series of Cubes, whoſe Roots are in arithmetical Progreſſion, beginning with a Cypher, *&c.* (as above) be infinitely continu'd, the Sum of all the Series will be one fourth NLLL = S.

That is, one fourth of the laſt Term ſo many Times repeated as there are Numbers of Terms.

Inſtances in Cube Numbers.

If 0, 1, 2, 3, 4, 5, *&c.* be the Roots of the Cubes.

$$1\left\{\frac{0+1+8+27}{27+27+27+27}=\frac{36}{108}=\frac{4}{12}=\frac{1}{4}\times\frac{1}{12}.\right.$$

$$2\left\{\frac{0+1\times 8\times 27\times 64}{64\times 64\times 64\times 64\times 64}=\frac{100}{320}=\frac{10}{32}=\frac{5}{16}=\frac{1}{4}\times\frac{1}{16}.\right.$$

$$3\left\{\frac{0\times 1\times 8\times 27\times 64\times 125}{125\times 125\times 125\times 125\times 125\times 125}=\frac{225}{750}=\frac{45}{150}=\right.$$

$$\left(\frac{15}{50}=\frac{1}{4}\times\frac{1}{20}\right.$$

From theſe Examples it plainly appears, that as the Number of Terms in the Series increaſes, the Fraction or Exceſs above one fourth decreaſes, the Exceſs being always $\frac{1}{4N-4}$, which, if we ſuppoſe the Series to be infinitely continu'd, will become infinitely ſmall, or nothing.

LEMMA V.

If a Series of Biquadrats, whoſe Roots are, in arithmetical Progreſſion, beginning with a Cypher, *&c.* as before, be infinitely continu'd, the Sum of all the Terms in ſuch a Series will be one fifth NLLLL.

The Truth of this may be manifeſt by the like Proceſs, as in the foregoing LEMMA's, and ſo on for higher Powers.

LEMMA

LEMMA VI.

The Sum of an infinite Progreffion, whofe greateft Term is a fquare Number, the others decreafing by odd Numbers, *viz.* 1, 3, 5, *&c.* is in fubfefquialteran Proportion of the Sum of the like Number of equal Terms, that is, as 2 to 3.

Inftances in fuch Progreffions.

$$1 \left\{ \frac{9+8+5}{9+9+9} = \frac{22}{27} = \frac{2}{3} + \frac{4}{9}. \right.$$

$$2 \left\{ \frac{16+15+12+7}{16+16+16+16} = \frac{50}{64} = \frac{2}{3} + \frac{11}{32}. \right.$$

$$3 \left\{ \frac{25+24+21+16+9}{25+25+25+25+25} = \frac{95}{125} = \frac{2}{3} + \frac{7}{75}. \right.$$

$$4 \left\{ \frac{36+35+32+27+20+11}{36+36+36+36+36+36} = \frac{161}{216} = \frac{2}{3} + \frac{17}{72}. \right.$$

From thefe Examples it plainly appears, that as the Number of Terms in the Series increafes, the Fraction or Excefs above $\frac{2}{3}$ decreafes; and if we fuppofe the Series to be infinitely continu'd, that Excefs will quite vanifh, and the Sum of the infinite Series will be $\frac{2}{3}$ of fo many equal to the greateft.

CHAP.

CHAP. II.

The Menſuration of SOLIDS.

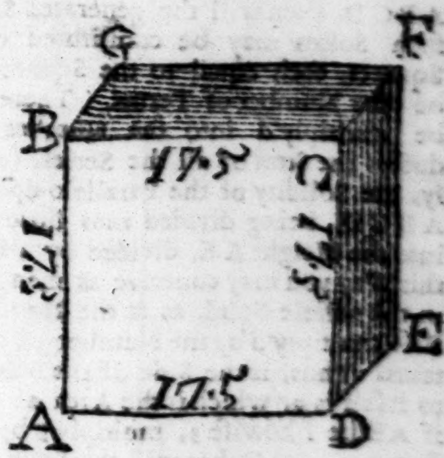
SOLID Bodies are ſuch as do conſiſt of Length, Breadth and Thickneſs; as Stone, Timber, Globes, Bullets, *&c.*

§ I. *Of a* CUBE.

A CUBE is a ſquare ſolid, comprehended under ſix geometrical Squares, being in the Form of a Dye. To find the ſolid Content, this is

The RULE.

Multiply the Side of the Cube into it ſelf, and that Product again by the Side; the laſt Product will be the Solidity, or ſolid Content of the Cube.




```
    17.5
    17.5
   -----
     875
   1225
   175
  ------
  306.25
    17.5
 -------
  153125
 214375
 30625
 -------
```

5359.375 the folid Content of the Cube.

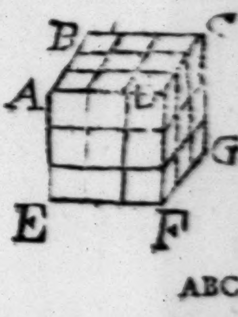
Suppofe ABCDEFG a cubical Piece of Stone or Wood, each Side thereof being 17 Inches and an half; multiply 17.5 by 17.5, and the Product is 306.25; which being multiplied by 17.5, the laft Product is 5359.375, which is 5359 folid Inches, and 375 Parts. To reduce the folid Inches to Feet, divide by 1728, (becaufe fo many cubical Inches is a Foot) and the folid Feet in the Cube will be 3, and .75 cubical Inches remain.

By Scale and Compaffes.

Extend the Compaffes from 1 to 17.5; that Extent, turn'd over twice from 17.5 will reach to 5359, the folid Content in Inches. Then extend the Compaffes from 1728 to 1, that Extent, turn'd the fame Way from 5359, will reach to 3.1 Feet.

DEMONSTRATION.

If the Square ABCD be conceiv'd to be mov'd down the Plain ADEF, always remaining Parallel to it felf, there will be generated, by fuch a Motion, a Solid having fix Plains, the two oppofite whereof will be equal and parallel to each other; whence it is call'd a Parallelopipedon, or fquare Prifm. And if the Plain ADEF be a Square equal to the generating Plain



ABCD,

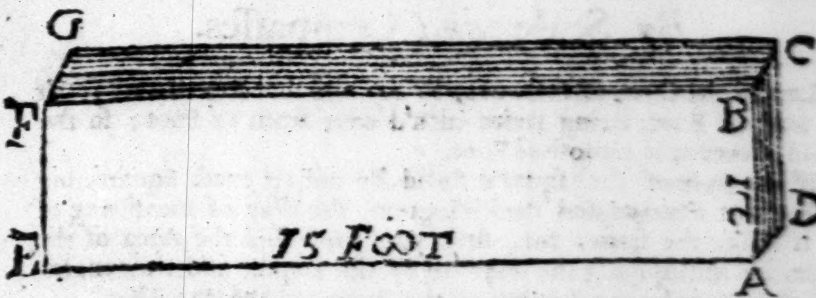
ABCD, then will the generated Solid be a Cube. From hence ſuch Solids may be conſtituted of an infinite Series of equal Squares, each equal to the Square ABCD; and AE or DF will be the Number of Terms. Therefore, if the Area of ABCD be multiply'd into the Number of Terms, AE, the Product is the Sum of all the Series, (*per* LEMMA I.) and conſequently, the Solidity of the Parallelopipedon or Cube. Or if the Baſe ABCD, being divided into little ſquare Area's, be multiply'd into the Height AE, divided by a like Meaſure for Length, after this Way you may conceive as many little Cubes to be generated in the whole Solid, as is the Number of the little Areas of the Baſe multiply'd by the Number of the Diviſions the Side AE contains. Thus, if the Side of the Baſe AB be 3, that multiply'd into it ſelf is 9, which is the Area of the ſquare Baſe ABCD; then, if AE be likewiſe 3, multiply 9 by 3, and the Product is 27; and ſo many little Cubes will this Solid be cut into, if you conceive it to be cut as the Lines direct.

From this Demonſtration it is very plain, that if you multiply the Area of the Baſe of any Parallelopipedon into its Length or Height, that Product will be the ſolid Content of ſuch a Solid.

§ II *Of a* PARALLELOPIPEDON.

LET ABCDEFG be a Parallelopipedon, or ſquare Priſm, repreſenting a ſquare Piece of Timber or Stone, each Side of its ſquare Baſe ABCD being 21 Inches, and its Length AE 15 Foot.

First,



Firſt, then, multiply 21 by 21, the Product is 441, the Area of the Baſe in Inches; which multiply'd by 180, the Length in Inches, and the Product is 79380, the ſolid Content in Inches. Divide the laſt Product by 1728, and the Quotient is 45.9, that is, 45 ſolid Feet, and 9 Tenths of a Foot. Or thus: Multiply 441 by 15 Feet, and the Product is 6615; divide this by 144, and the Quotient is 45.9, the ſame as before.

Or thus, by multiplying Feet and Inches.

Multiply 1 Foot 9 Inches by 1 Foot 9 Inches, and the Product is 3 Feet, 0 Inches, 9 Parts; this multiply'd again by 15 Feet, gives 45 Feet, 11 Inches, 3 Parts, that is, 45 Feet, and $\frac{11}{12}$ of a Foot, and $\frac{1}{4}$ of $\frac{11}{12}$.

See the Work of all theſe.

21		F. I.
21	441	1—9
	15	1—9
21		
42	2205	1—9
	441	1—3—9
441		
180	144)6615(45.9	3—0—9
		15
35280	855	
441	1350	45—0—0
		7—6
1728)79380(45.9	54	3—9
6912		
		45—11—3
10260		
8640		
16200		
15552		
648		

By

By Scale *and* Compaſſes.

Extend the Compaſſes from 12 to 21, and that Extent will reach to near 46 Foot, being twice turn'd over from 15 Foot; ſo the ſolid Content is almoſt 46 Foot.

If the Baſe of the ſquar'd Solid be not an exact Square, but in Form of a rectangled Parallelogram, the Way of meaſuring of it is much the ſame; for, firſt, you muſt find the Area of the Baſe, by multiplying the Breadth by the Depth, and then multiply that Area by the Length of the Piece, as before; Thus,

If a Piece of Timber be 25 Inches broad, 9 Inches deep, and 25 Feet long, how many Solid Feet are contain'd therein?

25	F. I.
9	2—1
	0—9
225	
25	1—6—9
	25
1125	
450	25—0—0
	12—6—0
144)5625(39	1—0—6
432	0—6—3
1305	39—0—9
1296	
09	
Anſwer 39 Feet.	

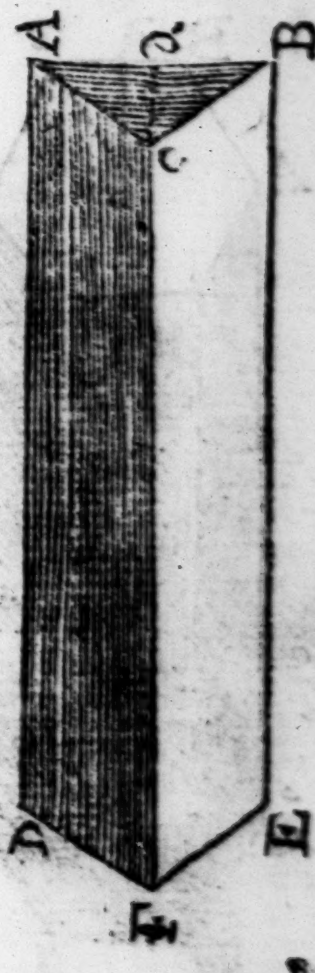
By Scale *and* Compaſſes.

Firſt, find a Mean geometrical, proportional between the the Breadth and the Depth; which to do upon the Line of Numbers, you muſt divide the Space upon the Line, between the Breadth and Depth, into two equal Parts; that middle Point will be the Mean proportional ſought; thus the middle Point between 25 and 9 is at 15, ſo is 15 a Mean proportional between 9 and 25; for, as 9 : 15 :: 15 : 25; ſo a Piece of Timber of 15 Inches ſquare, is equal to a Piece 25 Inches broad and 9 Inches deep. So then, if you extend the Compaſſes from 12 to 15, that Extent, turn'd twice over from 25 Feet, the Length, will reach to 39 Feet, the Content.

§. III.

§ III. *Of a Triangular* PRISM.

A Prism is a Solid contain'd under several Plains, and having its Bases like, equal, and parallel: The solid Content of a Prism (whether triangular or multangular) is found by multiplying the Area of the Base into the Length or Height, and the Product is the solid Content.



Let ABCDEF be a triangular Prism, each Side of the Base being 15.6 Inches, the Perpendicular thereof Ca is 13.51 Inches, and the Length of the solid 19.5 Feet.

Multiply the Perpendicular of the Triangle 13.51 by half the Side 7.8, and the Product is 105.378, the Area of the Base; which multiply by the Length 19.5, and the Product is 2054.871, which divide by 144; and the Quotient is 14.27 Feet, *fere*, the solid Content.

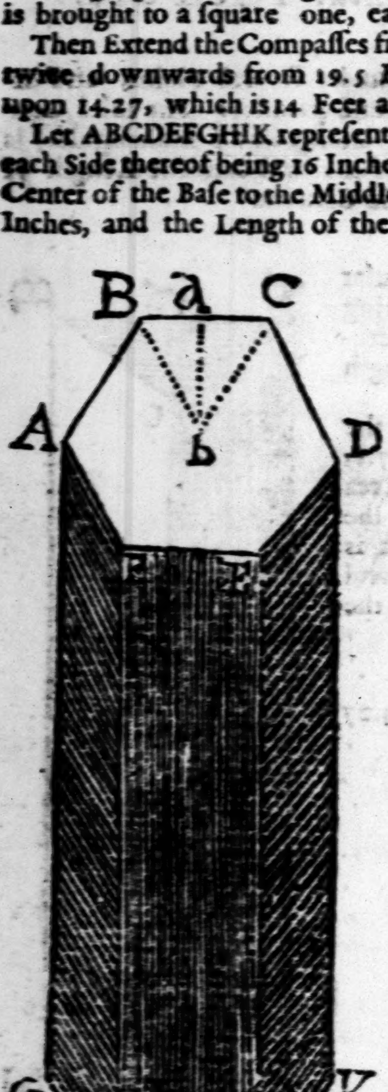
13.51	144)2054.87(14.27
7.8	144
10808	614
9457	576
105.378	388
19.5	288
526890	1007
948402	1008
105378	
2054.8710	

By Scale *and* Compaſſes.

Firſt, find a Mean proportional between the Perpendicular and half Side, (as before taught) by dividing the Space upon the Line, between, 13.51 and 7.8, into two equal Parts; ſo ſhall you find the middle Point between them to be at 10.26, which is the Mean proportional ſought: By this Means the triangular Solid is brought to a ſquare one, each Side being 10.26; Inches.

Then Extend the Compaſſes from 12 to 10.26; that Extent, turn'd twice downwards from 19.5 Feet, the Length, will at laſt fall upon 14.27, which is 14 Feet and a little above a Quarter.

Let ABCDEFGHIK repreſent a Priſm, whoſe Baſe is a Hexagon, each Side thereof being 16 Inches, and the Perpendicular, from the Center of the Baſe to the Middle of one of the Sides (a b) is 13.84 Inches, and the Length of the Priſm is 15 Feet, the Solid Content is requir'd.



Multiply half the Sum of the Sides 48 by 13.84, and the Product is 664.32, the Area of the Hexagonal Baſe, (by § 8. *p.* 86) which multiply by 15 Feet, the Length, the Product is 9964.8, which divided by 144, the Quotient will be 69.2 Feet, the ſolid Content requir'd.

13.84
48

11072
5536

664.32 Area of Baſe.
15

332160
66432

144)9964.80(69.2
864

1324
1296

288
288

By

By Scale *and* Compaſſes.

Firſt, find a Mean proportional between the Perpendicular, and half the Sum of the Sides, that is, divide the Space between 13.84 and 48, aud the middle Point will be 25.77 then extend the Compaſſes from 12 to 25.77 that Extent will reach (being twice turn'd over) from 15 Feet the Length, to 69.2 Feet the Content.

To find the ſuperficial Content of any of the forementioned'd Solids, you muſt take the Girt of the Piece, and multiply by the Length, and to that Product add the two Area's of the Baſes, the Sum will be the whole ſuperficial Content. Example of the Hexagonal Priſm laſt mention'd: The Sum of the Sides being 96, and the Length 15 Feet, that is, 180 Inches, which multiply'd by 96, the Product is 17280 ſquare Inches, to which add twice 664.32, the Area's of the two Baſes, and the Sum is 18608.64, the Area of the whole, which is 129.22 Feet.

```
   180          144)18608.64(129.22
    96              ————
  ————              420
  1080              1328
 1620                326
  ————                384
 17280               ————
   664.32              96
   664.32
  ————
```

The ſuperficial Content of the whole Solid is 129.22 Feet.

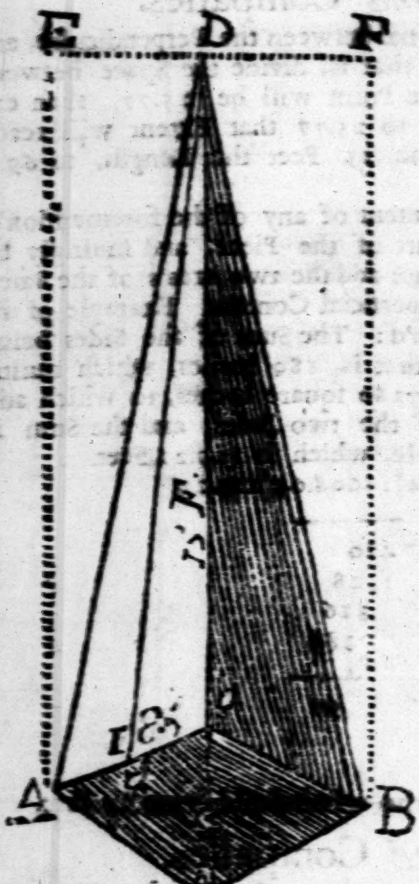
By Scale *and* Compaſſes.

Extend the Compaſſes from 144 to 180, that Extent will reach from 96 to 120 Feet. Then, to find the Area of the Baſe, extend the Compaſſes from 144 to 13.84, that Extent will reach from 48 to 4.6 Feet; add 120 Feet, and twice 4.6 Feet, and it makes 129.2 Feet, the ſuperficial Content, as before.

The Demonſtration of thoſe laſt Solids, will be the ſame as in firſt Section; for as in that, ſo in theſe, the Area of the Baſe is multiply'd into the Length to find the Content, and the ſame Reaſon is given for one as for the other.

§ IV. *Of a* PYRAMID.

A PYRAMID is a ſolid Figure, whoſe Baſe is a Polygon, and whoſe Sides are plain Triangles, their ſeveral Tops meeting together in one Point. To find the ſolid Content thereof, this is



The RULE.

Multiply the Area of the Base by a third Part of the Altitude, or Length, and the Product is the solid Content of the Pyramid.

Let ABD be a square Pyramid, each Side of the Base being 18.5 Inches, and the Perpendicular Height CD is 15 Feet: Multiply 18.5 by 18.5, and the Product is 342.25, the Area of the Base in Inches; which multiply'd by 5, a third part of the Height, and the Product is 1711.25; this divided by 144, the Quotient is 11.88 Feet, the solid Content.

```
   18.5
   18.5
  -----
   925
 1480
 185
  ------
 342.25  Area of the Base
      5
 ------
144)1711.25(11:88 Content.
```

F.	I.	Pts.	
1	6	6	
1	6	6	
	6	6	
	9	3	
		9	3
2	4	6	3
			5
1 10	7	3	

By

By Scale *and* Compaſſes.

Extend the Compaſſes from 12 to 18.5 Inches, that Extent turn'd twice over from 5 Feet, (a third Part of the Height) will fall at laſt upon 11.88 Feet, the ſolid Content.

To find the ſuperficial Content.

Multiply the ſlant Height (or Perpendicular of one of the Triangles) by half the Periphery of the Baſe 37, and the Product is 6668.88, which divided by 144, the Quotient is 46.31 Feet, the ſuperficial Content of all but the Baſe; then to that add 2.38 Feet, the Baſe, and it makes 48.69 Feet, the whole ſuperficial Content.

180.24 the ſlant Height dD
37
———
126168
54072
———
144)6668.88(46.31
576
———
908
864
———
448
432
———
168
144
———
24

46.31
2.38
———
48.69 the whole Content.

144)342.25(2.38
288
———
542
432
———
1105
1152

By Scale *and* Compaſſes.

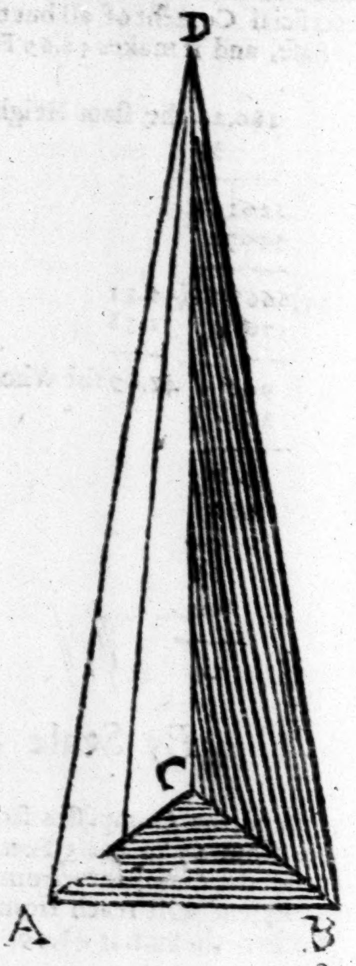
Extend the Compaſſes from 144 to 180.24, that Extent will reach from 37 to 46.31 Feet, the Area of the four Triangles; and extend the Compaſſes from 144 to 18.5 (one Side of the Baſe) that Extent will reach from 18.5 to 2.38, *fere*; which added to the other, the Sum is 48.69, the whole Superficies.

DEMONSTRATION.

Every Pyramid is a third Part of the Prism, that hath the same Base and Height, (by *Euc.* 12, 7.)

That is, the solid Content of the Pyramid ABD (in the last Figure) is one third Part of its circumscribing Prism ABEF.

For every Pyramid that hath a square Base, (such as AaBb in the last Figure) is constituted of an infinite Series of Squares, whose Sides or Roots are continually increasing in Arithmetical Progression, beginning at the Vertex or Point D, its Base AaBb being the greatest Term, and its perpendicular Height C D is the Number of all the Terms: But the last Term multiply'd into the Number of Terms, the Product will be triple the Sum of all the Series, (by LEMMA 3) consequently $\frac{NLL}{3} = S$. And S is equal to the solid Content of the Pyramid. From hence it will be easy to conceive, that every Pyramid is $\frac{1}{3}$ of its circumscribing Prism, (that is, of a Prism of equal Base and Altitude) what Form soever its Base is of, *viz.* whether it be square, triangular, pentangular, *&c.* You may very easily prove a triangular Pyramid to be a third Part of a Prism of equal Base and Altitude, by cutting a triangular Prism of Cork, and then cut that Prism into three Pyramids, by cutting diagonally, as I have several Times done, to satisfy my self and others.



Let ABCD be a triangular Pyramid, each Side of the Base being 21.5 Inches, and its perpendicular Height 16 Feet; the Content solid and superficial, is requir'd.

First, find the Area of the Base, by multiplying half the Side by the Perpendicular set fall from the Angle of the Base, to the opposite Side; which Perpendicular will be found to be 18.62; the Half thereof is 9.31, multiply'd by 21.5, the Product is 2000.165 Inches, the Area of the Base. Then, because the Altitude 16 cannot ex-

actly

actly be divided by 3, therefore I take the third Part of 200.165, which is 66.72, and multiply it by 16, and the Product is 1067.52, which divided by 144, the Quotient, is 7.41 Feet, the ſolid Content.

		F.	I.	Pts.	
9.31 half the Perpendicular.	Side	1	9	6	
21.5 the Side	half Perpend.		9	4	
4655					
931		1	4	1	6
1862				7	2
3)200.165 Area Baſe.	Area Baſe	1	4	8	8
					4
66.72 a third Part.					
16 Height.		5	6	10	8
					4
40032					
6672		3)22	3	6	8
144)1067.52(7.41 ſolid Cont.	Content.	7	5	2	2
1008					
595					
576					
192					
144					
48					

In caſting this up by Feet and Inches, inſtead of multiplying by 16, the Height, I break 16 into two ſuch Numbers, as being multiply'd together, the Product may be 16, *viz.* into 4 and 4, and multiply by firſt one, and then the other; a third Part of the laſt Product is the Content.

By Scale *and* Compaſſes.

Firſt, find a geometrical mean Proportional, (as before directed) by dividing the Space between 21.5 and 9.31 into two equal Parts, and you will find the middle Point at 14.15, which is the mean Proportional ſought. Then extend the Compaſſes from 12 to 14.15, that extent (turn'd twice over from 16 Feet) will fall at laſt upon 22.23 a third Part thereof is 7.41 Feet, the Content.

To find the ſuperficial Content.

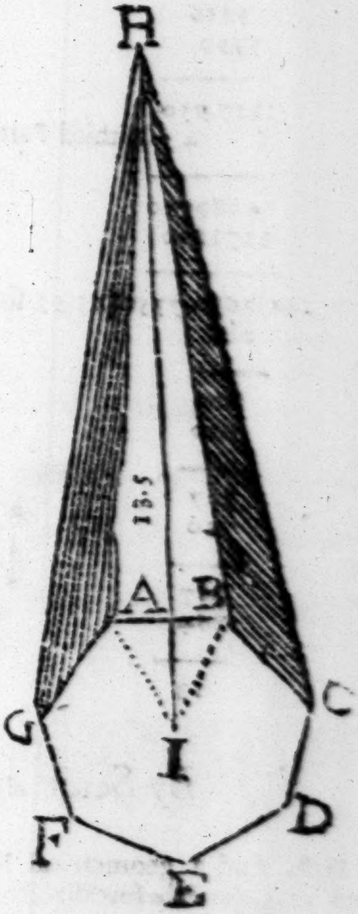
Multiply the ſlant Height (or Perpendicular of one of the Triangles) by half the Periphery of the Baſe, and to that Product add the Area of the Baſe, the Sum is the whole ſuperficial Content.

```
              192.1 Inches, the ſlant Height dD.
Half Periphery 32.25 = 21.5 + 10.75.
              ------
                9605
               3842
              3842
             5763
             ----------
             6195.225 Inches, the Area of all but the Baſe.
              200.165 Area of the Baſe add.
             ----------
         144)6395.390(44.41 Feet, the whole Content.
             576
             ---
              635
              576
              ---
              593
              576
              ---
               179
               144
               ---
                35
```

By Scale *and* Compaſſes.

Extend the Compaſſes from 144 to 192.1, that Extent will reach from 32.25 (half the Periphery of the Baſe) to 43.02 Feet, the Content of the upper Part.

And extend the Compasses from 144 to half the Perpendicular 9.31, that Extent will reach from the Side 21.5 to 1.39 Feet, the Area of the Base, which added to the other, makes 44.41 Feet, the Content of the whole.



Let A B C D E F G H be a Pyramid, whose Base is a Heptagon, each Side thereof being 15 Inches, and the Perpendicular of the Heptagon is 15.58 Inches, and the Perpendicular Height of the Pyramid H I is 13.5 Feet; the Content solid and superficial is requir'd.

Multiply 15.58 (the Perpendicular) by 52.5, half the Sum of the Sides of the Heptagon) and the Product is 817.95, which multiply'd by 4.5, *viz.* $\frac{1}{3}$ of the Height, and the Product is 3680.775.

Then divide this last Product by 144, and the Quotient is 25.56 Feet, the Content.

15.58 the Heptagon's Perpendicular.
52,5 the half Sum of the Sides.

7790
3116
7790

817.950
4.5 a third Part of the Height.

4089750
3271800

144)3680.7750(25.55 solid Feet.
288

800
720

807
720

877
864

13

By Scale *and* Compasses.

First, find a geometrical Mean proportional between 15.58 and 52.5, (as is before directed) which you will find to be 28.06; then extend the Compasses from 12 to 28.06 that Extent will reach from 4.5 (twice turn'd over) to 25.56 Feet.

To find the superficial Content.

Multiply the Height taken from the Middle of one of the Sides of the Base 162.75 Inches, by the half Sum of the Sides 52.5 Inches, and the Product is 8544.375; which divided by 144, the Quotient is 59.335 Feet, the Content of the upper Part.

162.75

```
        162.75                              144)817.95(5.68
          52.5                                   979
        ------                                   1155
         81375                                  -----
        32550                                      3
       81375
      -------
144)8544.375(59.335 Feet.
    ----     5.68 Baſe add.
 1344       ------
  483       65.015 the whole Content.
   517
   855
  -----
   135
```

By Scale *and* Compaſſes.

Extend the Compaſſes from 144 to 162.75, that Extent will reach from 52.5 to 59.335 Feet.

And extend the Compaſſes from 144 to 15.58, the Perpendicular of the Heptagon, the Extent will reach from 52.5 to 5.68 Feet the Content of the Baſe, which add to the former, the Sum is 65.015, the whole ſuperficial Content.

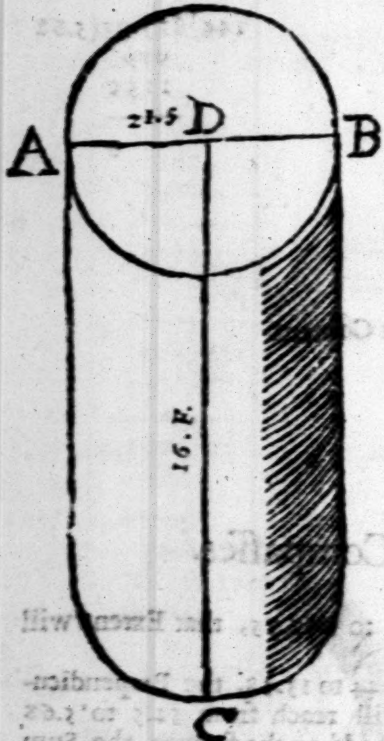
§ V. *Of a* CYLINDER.

A CYLINDER is a round Solid, having its Baſes circular, equal, and parallel, in Form of a Rolling-Stone uſed in Gardens. To find the ſolid Content thereof, this is

The RULE.

Multiply the Area of the Baſe by the Length, and the Product is the ſolid Content.

Let



Let A B C by a Cylinder, whose Diameter A B is 21.5 Inches, and the Length C D is 16 Foot, the solid Content is requir'd.

First, square the Diameter 21.5, and it makes 462.25; which multiply by .7854, and the Product is 363.05115. Then multiply this by 16, and the Product is 5808.8164. Divide this last Product by 144, and the Quotient is 40.34 Feet, the solid Content.

By Scale *and* Compasses.

Extend the Compasses from 13.54 to 21.5, the Diameter, that Extent (turn'd twice over from 16, the Length) will at last fall upon 40.34, the solid Content.

To find the superficial Content.

First, (by Chap. I. Sect. IX. Prob. 2.) find the Circumference of the Base 67.54, which multiply by 16, the Product is 1080.645 which divided by 12, the Quotient is 90.05 Feet, the Curve-Surface; to which add 5.04 Feet, the Sum of the two Bases, and the Sum is 95.09 Feet, the whole superficial Content.

```
   67.54                              363.0
      16                                  2
 -------                           --------
   40524                          144)726. 0(5.04
   6754                              ------
 -------          90.05 } add           6 0
12)108064          5.04 }
 -------         ------
   90.05          95.09
```

By

By Scale *and* Compaſſes.

Extend the Compaſſes from 12 to 67.54, (the Circumference) that Extent will reach from 16 (the Length) to 90.05 Feet, the Curve-Surface.

And extend the Compaſſes from 12 to 21.5, (the Diameter) that Extent (turn'd twice from .7854) will at laſt fall upon 2.52 Feet, the Area of one Baſe; which doubled is 5.04; this added to the Curve-Surface, makes 95.09 Feet, the whole ſuperficial Content.

DEMONSTRATION.

The ſolid Content of every Cylinder is found, by multiplying the Area of its Baſe into its Height, as aforeſaid: For every right Cylinder is only a round Priſm, being conſtituted of an infinite Series of equal Circles; that of its Baſe, or End, being one of the Terms, and its Height CD (in the former Figure) is the Number of all the Terms. Therefore the Area of its Baſe AB being multiply'd into CD, will be its Solidity, (by Lemma I.) Let D = AB, and H = CD.

Then .7854 DD x H = its Solidity.

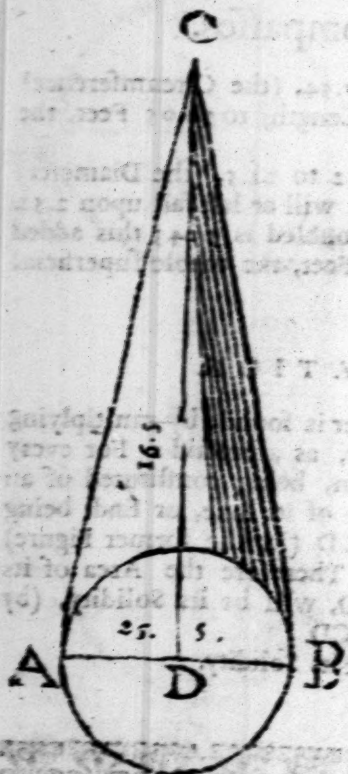
§ VI. *Of a* CONE.

A Cone is a Solid, having a circular Baſe, and growing ſmaller and ſmaller, 'till it ends in a Point, which is call'd the Vertex, and may be nearly repreſented by a Sugar-Loaf. To find the Solidity thereof, this is

The RULE.

Multiply the Area of the Baſe by a third Part of the Perpendicular Height, and the Product is the ſolid Content.

Let



Let ABC be a Cone, the Diameter of whose Base AB is 26.5 Inches, and the Height of the Cone DC is 16.5 Feet: First, square the Diameter 26.5, and it is 702.25; which multiply by .7854, and the Product is 551.547155 which multiply by 5.5, and the Product is 3033.478255; which divided by 144, the Quotient is 21.07, *fere*, the solid Content of the Cone.

26.5 the Diameter.
26.5

1325
1590
530

702.25 the Square.
.7854

280900
351125
561800
491575

551.54|715 Area of the Base.
5.5 a 3d part of the Height.

275773
275773

144)3033.503(21.07 Feet, the Content.

153
947

By Scale and Compasses.

Extend the Compasses from 13.54 to 26.5 (the Diameter) that Extent turn'd twice over from 5.5, (a third part of the Height) will at last fall upon 21.07 Feet, the Content.

Th

To find the ſuperficial Content.

Multiply half the Circumference 41.626 hy the ſlant Height AC 198.46, the Product is 8261.09526; which, divided by 144, the Quotient is 57.37, *fere*, the Curve-Surface; to which add the Baſe, the Sum is 61.2 the ſuperficial Content.

```
   41.626 the half Circumference of the Baſe.
   198.46 the ſlant Height.
   ------
   249756
  166504
 333008
374634
41626
------------
144)8261.09596(57.37 Feet, fere.
    ----        3.83 the Baſe add
   1061        -----
     530       61.20 the whole Content.
      989
144)551.54(3.83
    -----
    1195
     434
    -----
       2
```

By Scale *and* Compaſſes.

Extend the Compaſſes from 144 to 198.46, that Extent will reach from 41.626 to 57.37 Feet, the Curve-Surface.

And extend the Compaſſes from 12 to 26.5 (the Diameter) that Extent, turn'd twice over from .7854, will at laſt fall upon 3.83 Feet, the Baſe; which added to 57.37, the Sum is 61.2 Feet. the ſuperficial Content.

DEMONSTRATION.

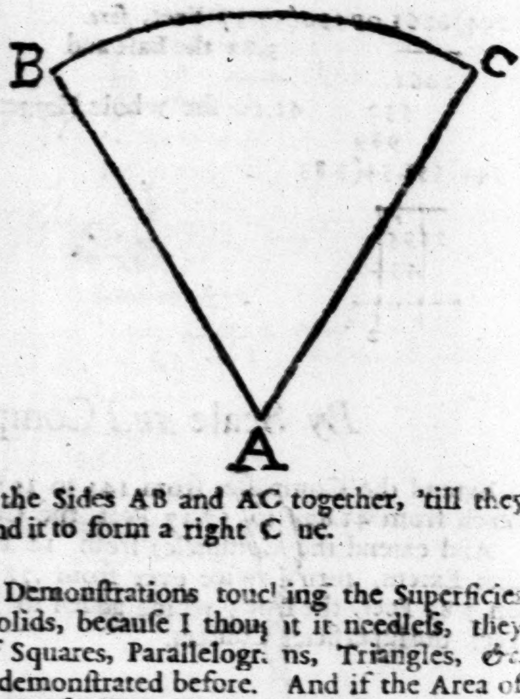
Every Co[illegible] third Part of a Cylinder of equal Baſe and Altitude. The Truth of this may eaſily be conceiv'd, by only conſidering, that a Cone is but a round Pyramid, and therefore it

it muſt needs have the ſame Ratio to its circumſcribing Cylinder, as the ſquare Pyramid hath to its circumſcribing Parallelopipedon, *viz.* as 1 to 3. However, to make it yet clearer, let it be farther conſider'd, That,

Every right Cone is conſtituted of an infinite Series of Circles, whoſe Diameters do continually increaſe in Arithmetical Progreſſion, beginning at the Vertex, or Point C, the Area of its Baſe AB being the greateſt Term, and its perpendicular Height DC, the Number of all the Terms; therefore the Area of the Circle of the Baſe, multiply'd by a third Part of the Altitude DC, will be rhe Sum of all the Series, equal to the Solidity of the Cone, by LEMMA III.

The Curve-Superficies of every right Cone, is equal to half the Rectangle of the Circumference of its Baſe into the Length of its Side.

For the Curve-Surface of every right Cone, is equal to the Sector of a Circle, whoſe Arch BC is equal to the Periphery of the Baſe of the Cone, and Radius AB equal to the ſlant Side of the Cone. Which will appear very evident, if you cut a Piece of Paper in the Form of a Sector of a Circle, as ABC, and bend both the Sides AB and AC together, 'till they meet, and you will find it to form a right C ne.



I have omitted the Demonſtrations touc' ing the Superficies of all the foregoing Solids, becauſe I thou t it needleſs, they being all compos'd of Squares, Parallelogr. ns, Triangles, *&c.* which Figures are all demonſtrated before. And if the Area of all ſuch Figures as compoſe the Solid, be und ſeverally, and added together, the Sum will be the ſuper ial Content of the Solid.

§ VII. *Of*

§ VII. *Of the Fruſtrum of a PYRAMID.*

A FRUSTRUM of a Pyramid, is the remaining Part, when the Top is cut off by a Plane parallel to the Baſe. To find the ſolid Content thereof, there are ſeveral Rules.

RULE I.

To the Rectangle (or Product) of the Sides of the two Baſes add the Sum of their Squares; that Sum being multiply'd into one third Part of the Fruſtrum's Height, will give its Solidity, if the Baſes be ſquare.

Or thus, which is the ſame in Effect.

Multiply the Area's of the two Baſes together, and to the ſquare Root thereof add their two Area's; that Sum, multiply'd by one Third of the Height, gives the Solidity of any Fruſtrum, ſquare or multangled.

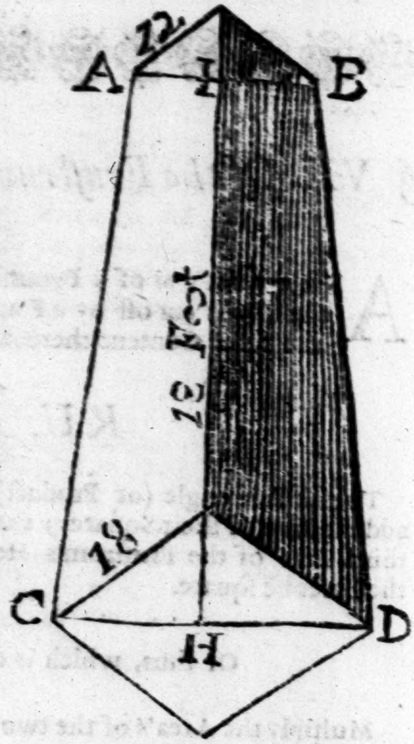
RULE II.

To the Rectangle of the Sides of the two Baſes, add one third Part of the Square of their Difference; that Sum being multiply'd into the Height, will produce the Solidity, if the Baſes be Squares: But if they be triangu r or multangular, the Rectangle of the Sides, with the third P rt of the Square of their Difference, will be the Square of a m an Side; and the ſquare Root thereof will be ſuch a mean Side s will reduce the tapering Solid to a Priſm equal thereunto.

Example. Let A CD be the Fruſtrum of a ſquare Pyramid, the Side of th grea Baſe 18 Inches, and the Side of the leſſer 12 Inches, he Height 18 Foot; the Solidity thereof is required.

Firſt,

First, multiply the two Sides together 18 by 12, and the Product is 216, and the Difference of the Sides is 6, whose Square is 36; a third Part thereof is 12, which added to 216, the Sum is 228 Inches, the Area of a mean Base; which multiply'd by 18 Feet, the Length, the Product is 4104; this divided 144, the Quotient is 28.5 Feet, the Content.



Or, by the first Rule, thus; the Square of 18 is 324, and the Square of 12 is 144, and the Rectangle of 18 by 12 is 216; the Sum of these three 684, which multiply'd by 6, the Product is 4104; which divided by 144, the Quotient is 28.5 Feet, the same as before.

See the WORK of both Ways.

18	6 Diff.	18	12
12	6	18	12
216	3)36 Square	324 Square	144 Square.
12 add	12 a Third.	144	
228 the Sum		216	
18 the Height		684 the Sum.	
1824		6 a 3d of the Height.	
228		144)4104(28.5 Feet.	
144)4104(28.5		1224	
1224		720	
720		...	
...			

By

By Feet and Inches thus:

	F.	I.		I.
Multiply	1	6		6
by	1			6
Product	1	6		3)36 q
add	0	1		12
Multiply	1	7		
by	18	0	Height.	
	18	0		
	9	0		
	1	6		
Content	28	6		

Or thus,

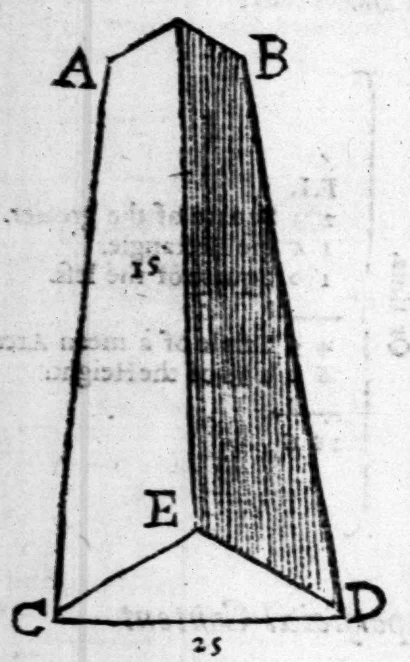
F.	I.	
2	3	Square of the greater.
1	6	the Rectangle.
1	0	Square of the leſs.
4	9	Triple of a mean Area.
6	0	a 3d of the Height.
28	6	

To find the ſuperficial Content.

The Perimeter of the greater Baſe is 72, and the Perimeter of the leſſer Baſe is 48; add both the Perimeters together, the Sum is 120; the Half thereof is 60; which multiply'd by 18 Feet, the Product is 1080; this divided by 12, the Quotient is 90 Feet; to which add the two Baſes 2.25 Feet, and 1 Foot, the Sum is 93.25 Feet, the whole ſuperficial Content.

18	12	18	the Height.
4	4	60	
72	48	12)1080	
48		90	Feet.
		2.25	the greater Baſe.
2)120		1	the leſſer Baſe.
60		93.25	Sum.

L Again



Again, Let A B C D be the Fruſtum of a triangular Pyramid, each Side of the greater Baſe 25 Inches, and each Side of the leſſer Baſe 9 Inches, and the Length 15 Feet, the ſolid Content thereof is requir'd.

By the ſecond Rule, multiply 25 by 9, and the Product is 225; and the Difference between 25 and 9 is 16; which ſquar'd, makes 256, a third Part thereof is 85.333, which added to 225, and the Sum is 310.333, this multiply'd by .433, the Product is 134.374, &c. which is the Area of a mean Baſe; and that multiply'd by 15 Feet, the Length, the Product is 2015.610; which divided by 144, the Quotient is 13.99 Feet, the Solidity.

Or thus, by the latter Part of the firſt Rule: Find the Area of the greater Baſe, which you will find to be 270.625. and the Area of the leſſer Baſe will be 35.073; theſe two Area's multiply'd together, the Product is 9491.630625, the ſquare Root thereof is 97.425; to which add the two Area's, and the Sum is 403.123; which multiply'd by a third Part of the Length 5, and the Product is 2015.615; and that divided by 144, and the Quotient is 13.99 Feet, as before.

See

See the Working of both.

```
         25    25
          9     9
      -----   ---
Product 225    16 Difference.
               16
              ---
               96
              16
           ------
           3)256 the Square.
           -----
            85.333 a third Part.
           225.    add.
           -------
           310.333
              .433 Tabular Number (vide p. 87.)
           --------
           930999
          930999
         1241332
         ---------
         134.37418 9 mean Area.
                 15 Length.
         ---------
         671870945
        134374189
      ----------------
144)2015.61|2835(13.99 Feet.
      -------
      575
     1436
       1401
     -----
      105
```

```
  25
  25
 ----
 125
 50
 ----
 625 Square.
 .433
 ----
  1875
 1875
2500
------
270.625 Area.
```

```
 9
 9
---
81 Square.
```

```
 .433
  81
-----
  433
 3464
-----
35.073

 270.625
  35.073
---------
  811875
 1894375
1353125
811875
------------
9491.630625(97.425
81
----
187)1391
    1309
   -----
1944) 8263
      7776
     -----
19482) 48706
       38964
      ------
194845) 974225
        974225
       -------
```

270.625	greater Area.
97.425	the mean Proportional.
35.073	the leſſer Area.
403.123	the Triple of a mean Area.
5	a third Part of the Height.

144) 2015.615(13.99 Feet, the Solidity.
575
1436
1401
―――
105

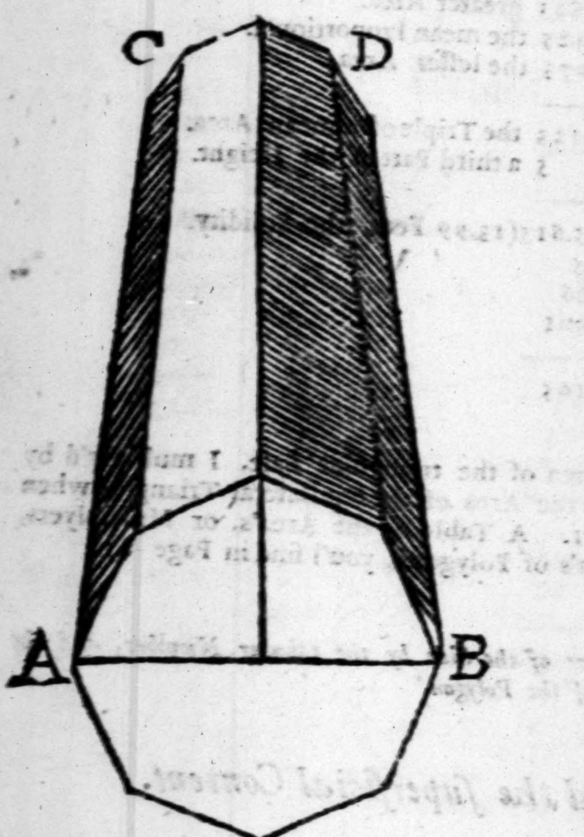
In finding the Area of the triangular Baſe, I multiply'd by .433, becauſe that is the Area of the equilateral Triangle, when the Side thereof is 1. A Table of the Area's, or Multiplyers, for finding the Area's of Polygons, you'l find in Page 87.

Multiply the Square of the Side by the tabular Number, and the Product is the Area of the Polygon.

To find the ſuperficial Content.

The Perimeter of the greater Baſe is 75, and the Perimeter of the leſſer Baſe is 27; the Sum of both is 102, and the half Sum is 51; which multiply'd by 15 Feet, the Product is 765; which divided by 12, the Quotient is 63.75; to which add the Sum of the two Baſes 2.12 Feet, and the Sum is 65.87 Feet, the whole ſuperficial Content.

NOTE, That 51 ſhould have been multiply'd by the ſlant Height, but the Difference it would make, is but .06 of a Foot, which is inconſiderable.



Again, Suppoſe ABCD to be the Fruſtum of a Pyramid, having an Octagonal Baſe, each Side thereof being 9 Inches, and each Side of the leſſer Baſe 5 Inches, and the Length or Height 10.5 Feet, the Solidity is requir'd.

By the ſecond Rule, Multiply the greater Side 9, by the leſſer Side 5, and the Product is 45; then the Difference between 9 and 5 is 4, which ſquar'd, makes 16; a third Part thereof is 5.3333, which added to 45, the Sum is 50.3333; multiply this laſt by the Number in the Table 4.8284, and the Product is 243.0292, the Area of a mean Baſe; which multiply'd by the Height 10.5 Feet, and the Product is 2551.8066; then divide this laſt Product by 144, and the Quotient is 17.72 Feet, the ſolid Content.

See the WORK

Mult.

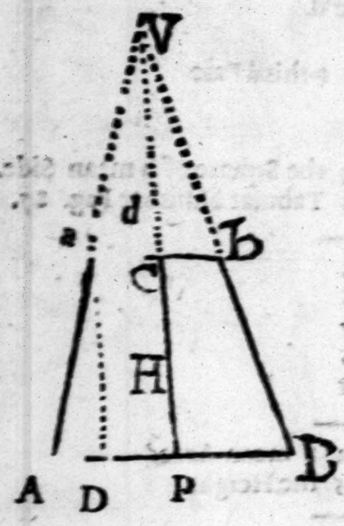
```
Mult. 9 Inches.        9 from the greater Side
   by 5 Inches.        5 ſubtract the leſſer.
      ——              ——
Prod. 45               4
                       4
                      ——
                     3)16 ſquar'd.
                      ——
                     5.3333 a third Part
              Add 45
                     ————
              Sum 50.3333 the Square of a mean Side.
                   4828.4 Tabular Number Pag. 87.
                 ————————
                  2013332
                   402666
                    10067
                     4026
                      201
                 ————————
                 243.0292 a mean Area.
                     10.5 the Height.
                 ————————
                 12151460
                2430292
               ————————————
           144)2551.80|660(17.72
               144
               ———
               1111
               1008
               ————
               1038
               1008
               ————
                300
                288
                ———
                 12
```

To find the ſuperficial Content.

The Perimeter of the greater Baſe is 72, and the Perimeter of the leſſer Baſe is 40, and their Sum is 112; the half thereof is 56, which multiply'd by the Height 10.5 Feet, and the Product is 588; which divided by 12, the Quotient is 49 Feet; to which add the Sum of the two Faces, 3.55, and the Sum is 52.55 Feet, the whole ſuperficial Content.

DEMONSTRATION.

From the Rules deliver'd in the IVth and VIth Sections, the two foregoing Rules may eaſily be demonſtrated.



Suppoſe a Square Pyramid, as ABV, to be cut by a Plain at a b, parallel to its Baſe A B, and it were requir'd to find the Solidity of the Fruſtum, or Part a b A B. Let there be given.

D = BA, the Side of the greater Baſe.
d = ba, the Side of the leſſer Baſe.
H = CP, the perpendicular Height.

First,	1	$D - d : H :: d : \frac{dH}{D - d} =$ VC by the Figure.
Then	2	$DD \times \frac{H + VC}{3} =$ the whole Pyramid BVA by Section the IVth.
And	3	$dd \times \frac{1}{3} VC =$ the Pyramid a b cut off.

Then, in the 2d and 3d Steps, if inſtead of VC, you take $\frac{dH}{D - d}$ equal to it by the firſt Step, it will be,

viz. 1.2.	4	$\frac{DDDH}{3D - 3d} =$ the whole Pyramid BVA.
and 1.3	5	$\frac{dddH}{3D - 3d} =$ the Pyramid aVb.
4—5.	6	$\frac{DDDH - dddH}{3D - 3d} =$ the Fruſtum abAB.

And by dividing DDD—ddd by D—d, and then multiplying the Quotient by $\frac{1}{3}$H, the laſt Step will be reduc'd to DD+Dd +dd : x$\frac{1}{3}$H = the Fruſtum abAB, which in Words is thus;

To the Rectangle of the Sides of the two Baſes add the Sum of their Squares; that Sum being multiply'd into one Third of the Fruſtum's Height, will give its Solidity, which is the ſame as the firſt Rule of this Section.

See the WORK of the DIVISION.

```
D—d)DDD—ddd(DD+Dd+dd
    DDD—DDd
    ————————
        Ddd—ddd
        Ddd—ddd
        ———————
           Ddd—ddd
           Ddd—ddd
           ———————
             0  0
```

The ſame Reaſon will hold good for all Fruſtums of Pyramids or Cones, whether the Baſe be triangular or multangular, becauſe the Squares of the Sides of any Figure, or the Squares of the Diameters of Circles, are proportional to the Area, which proves the latter Part of the ſaid firſt Rule.

Again.

Again, to prove the ſecond Rule.

Suppoſe	1	$x = D - d$. And $F =$ the Fruſtum.
then	2	$DD + Dd + dd = \frac{3F}{H}$ by the laſt.
1 ⊙ 2	3	$xx = DD - 2Dd + dd$.
2 — 3	4	$3Dd = \frac{3F}{H} - xx$.
4 ÷ 3	5	$Dd = \frac{F}{H} - \frac{1}{3}xx$. Or $Dd + \frac{1}{3}xx = \frac{F}{H}$
5 × H	6	$Dd + \frac{1}{3}xx : \times H = F$, the Fruſtum abAb.

Which in Words is thus:

To the Rectangle of the Sides of the two Baſes add one third Part of the Square of the Difference of the ſaid Sides, and multiply the Sum by the Height of the Fruſtum, the Product is the Solidity of the Fruſtum.

The ſuperficial Contents of Fruſtums, (all but the Baſes) are compos'd of Trapeziums, ſo many as the Fruſtum has Sides. As the ſquare Fruſtum abAB in the laſt Figure is compos'd of four Trapeziums, having the two upper, and alſo the two lower Angles equal; if therefore the Trapezium abAB be cut in two by the Line CP, and the two Pieces laid together, the Line bB upon the Line aA, the narrow End of the one to the broad End of the other, it will form a right-angled Parallelogram, as is plain by the Figure annex'd; the Parallelogram DCEP being equal to the Trapezium abAB; becauſe the Side Da is equal to PB, and EA is equal to aC. Therefore, to find the Area of the Trapezium, add half the Side ab to half the Side AB, and it makes DC or EP; which multiply by the Height PC, the Product is the Area of the Parallelogram DCEP, equal to the Trapezium; abAB; then, if that be multiply'd by the Number of Trapeziums, the Product will be the ſuperficial Content of the

Fruſtum;

Fruſtum, wanting the Baſes. Or if the whole Perimeter, of the greater Baſe be added to the Perimeter of the leſſer Baſe, and half the Sum multiply'd by the Height, the Product will be the ſuperficial Content of all the Trapeziums at once.

NOTE, That half the Sum of the Perimeters, ſhould be multiply'd by the ſlant Height, up the Middle of one of the Trapeziums; but in the foregoing Examples I have multiply'd by the Perpendicular Height, becauſe the Difference is very inconſiderable.

§ VIII. *Of the Fruſtum of a CONE.*

A FRUSTUM of a CONE, is that Part which remains, when the Top-End is cut off by a plane Parallel to the Baſe. To find the ſolid Content, the Rules are the ſame in Effect as for the Fruſtum of a Pyramid.

RULE I.

To the Rectangle of the Diameters of the two Baſes add the Squares of the ſaid Diameters, and multiply the Sum by .7854. the Product will be the Triple of a mean Area; which multiply'd by $\frac{1}{3}$ of the Perpendicular Height, that Product will be the ſolid Content.

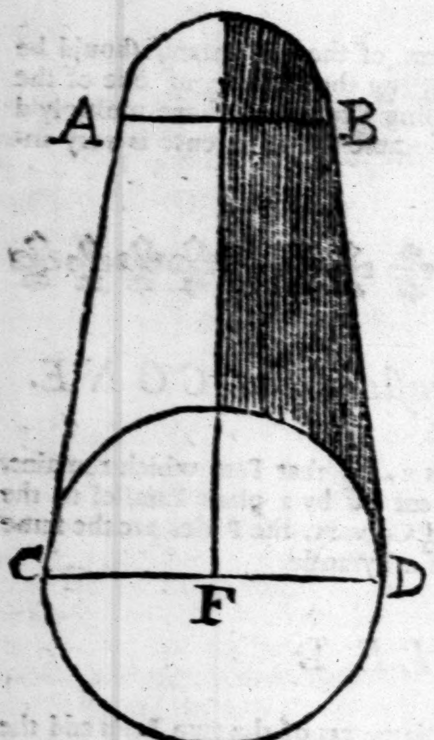
Or thus: Multiply the Area's of the greater and leſſer Baſes together, and out of the Product extract the ſquare Root, and add the two Area's and ſquare Root together, and multiply the Sum by one Third of the Perpendicular Height, the Product is the ſolid Content.

RULE II.

To the Rectangle of the greater and leſſer Diameters, add one third Part of the Square of their Difference, and multiply

the

the Sum by .7854, the Product is a mean Area; which multiply'd by the Perpendicular Height, the Product is the Solidity.



Example. Let ABCD be the Fruſtum of a Cone, whoſe greater Diameter C D is 18 Inches, and the leſſer Diameter AB 9 Inches, and the Length 14.25 Feet, the ſolid Content is requir'd.

Multiply 18 by 9, and the Product is 162, and the Difference between 18 and 9, is 9, whoſe ſquare is 81, a third Part is 27; which add to 162, the Sum is 189; this Multiply'd by .7854, the Product is 148.44; which divided by 144, the Quotient is 1.03 Feet, the Area of a mean Baſe; which multiply by 14.25 Feet, the Height, the Product is 14.6775 Feet, the Solid Content.

Or thus, by the firſt Rule.

The Square of 18 (the greater Diameter) is 324, and the Square of 9 (the leſſer Diameter) is 81, and the Rectangle, or Product of 18 by 9, is 162; the Sum o theſe three is 567, which multiply'd by .7854, the Product is 445.3218; which divided by 144, the Quotient is 3.09 Feet, the triple Area of a mean Baſe; this multiply'd by 4.75 Feet, (a third Part of the Height) and the Product is 14.6775 Feet, the Solidity, the ſame as before.

See

See the WORK.

18	18 from	.7854
9	9 fubtr.	189
162	9 rem.	70686
Add 27	9	62832
		7854
Sum 189	3)81 Square	
		144)148.44|06(1.03
	27 a Third.	144
		444
		432
		12

Height 14.25 Feet.
Area Bafe 1.03 Feet.

4275
1425

Solid Content 14.6775 Feet.

324 the Square of 18.
162 the Rectangle.
81 the Square of 9.

567 the Triple Square of a mean Diameter.

.7854
567

54978
47124
39270

144)445.32|18 (3.09
4.75 a third of the Height

1332

36

1545
2153
1236

The Solidity 14.6775

To

To find the superficial Content.

By Chap. I. Sect. IX. *Problem* 2. you will find the Circumference of the greater Base to be 56.5488, and of the lesser Base 28.2744; the Sum of both is 84.8232; the half Sum is 42.4116; which multiply'd by 14.25 Feet, and the Product is 604.36, *&c.* which divided by 12, the Quotient is 50.36 Feet, the Curve-Surface; to which add the Sum of the two Bases, 2.21 Feet the Sum is 52.57 Feet, the whole superficial Content.

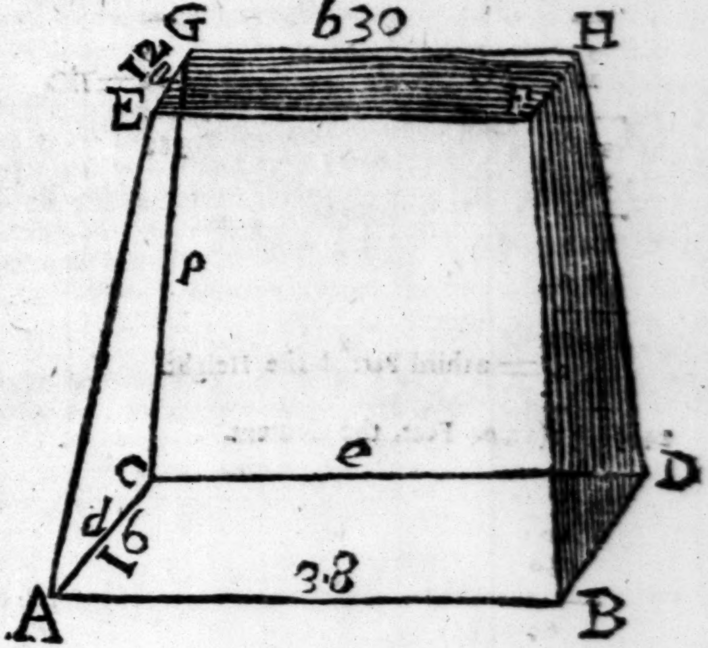
§ IX. *To measure the* Frustum *of a rectangled* Pyramid, *call'd a* PRISMOID, *whose Bases are Parallel one to another, but disproportional.*

The RULE.

TO the greatest Length add half the lesser Length, and multiply the Sum by the Breadth of the greater Base, and reserve the Product.

Then, to the lesser Length, add half the greater Length, and multiply the Sum by the Breadth of the lesser Base; and add this Product to the other Product reserv'd, and multiply that Sum by a third Part of the Height, and the Product is the solid Content.

Example.



Example. Let ABCDEFGH be a Prismoid given, the Length of the greater Base AB 38 Inches, and its Breadth AC 16 Inches; and the Length of the lesser Base EF is 30 Inches, and its Breadth 12 Inches, and the Height 6 Feet; the solid Content is requir'd.

To the greater Length AB 38, add half EF the lesser Length 15, the Sum is 53; which multiply'd by 16, the greater Breadth, and the Product is 848; which reserve.

Again, To EF 30, add half AB 19, and the Sum is 49; which Multiply by 12, (the lesser Breadth EG) the Product is 588; to which add 848, (the reserv'd Product) and the Sum is 1436; which multiply'd by 2, (a third Part of the Height) and the Product is 2872; divide this Product by 144, and the Quotient is 19.94 Feet, the solid Content.

38 = AB	30 = EF
15 = half EF	19 = half AB
53	49
16 = AC	12 = EG
318	588
53	

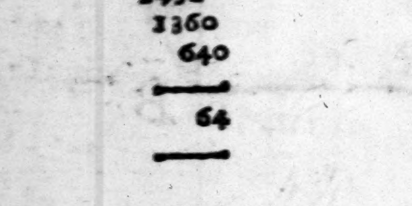
848
588

1436
2 = a third Part of the Height.

144)2872(12.94 Feet, the Content.
1432
1360
640
64

To prove this RULE.

Let us suppose the Solid cut into Pieces, so as to make it capable of being measur'd by the foregoing Rules, thus; Let ABCD represent the greater Base, and EFGH the lesser Base; and let the Solid be suppos'd to be cut thro' by the Lines ac, bd, and ef, gh, from the Top to the Bottom, so will there be a Parallelopipedon, having its Bases equal to the lesser Base EFGH, and its Height 6 Feet, equal to the Height of the Solid: Multiply 30 (the Length of the Base) by 12, (the Breadth thereof) and the Product is 360; which multiply'd by the Height 6 Feet, and the



the Product is 2160. Then there are two Wedge-like Pieces, whose Bases are abEF, and GHcd; if these two Pieces be laid together, the thick End of one to the thin End of the other, they will compose a rectangled Parallelopipedon; which to measure, multiply the Length of the Base 30 by its Breadth 2, and the Product is 60; which multiply'd by 6, (the Height) the Product is 360. Then there are two other Wedge-like Pieces, whose Bases are eEgG, and FfHh; these two laid together, will compose a rectangled Parallelopipedon; to measure this, multiply the Length of the Base 12 by the Breadth 4, the Product is 48; which multiply'd by 6, (the Height) the Product is 288. And lastly, there are four rectangled Pyramids, at each Corner one; which to measure, multiply the Length of one of the Bases 4, by its Breadth 2, the Product is 8; which multiply'd by 2, (a third Part of the Height) the Product is 16; and that multiply'd by 4, (because there are four of them) and the Product is 64. Then add all these together, and the Sum is 2872; and divide by 144, the Quotient is 19.94 Feet, the same as before, which shews the Rule to be true.

See the WORK.

12	30	12	4
30	2	4	2
360	60	48	8
6	6	6	2
2160	360	288	16
360			4
288			64
64			

144)2872(19.94 Feet, the whole Content.

1432
1360
640

64

To find the superficial Content.

Half the Perimeter of the greater Base is 54, and half the Perimeter of the lesser Base is 42; which added together, the Sum is 96; which multiply'd by 6, (the Height) the Product is 576: Divide this Product by 12, the Quotient is 48 Feet; to which add the Sum of the two Bases 6.72 Feet, and the Sum is 54.72 Feet, the whole superficial Content.

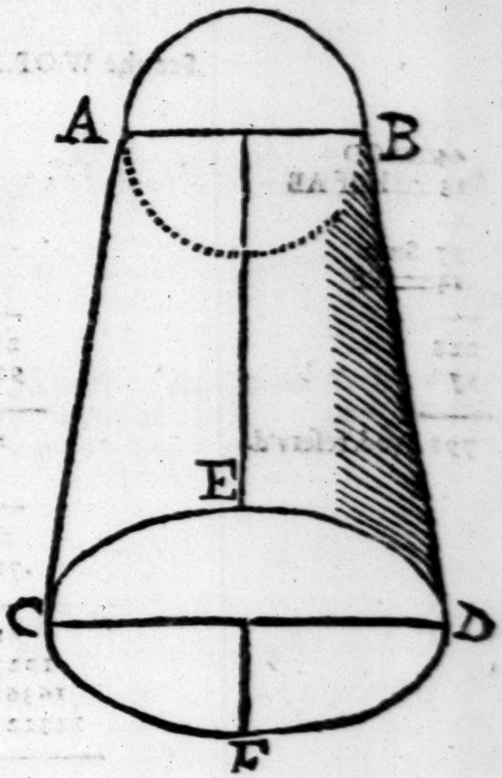
§ X. *To measure a* CYLINDROID; *that is, a* Frustum *of a* CONE *having its Bases Parallel to each other, but unlike.*

The *RULE*.

TO the longest Diameter of the greater Base add half the longest Diameter of the lesser Base, and multiply the Sum by the shortest Diameter of the greater Base, and reserve the Product.

Then, to the longest Diameter of the lesser Base add half the longest Diameter of the greater Base, and multiply the Sum by the shortest Diameter of the lesser Base and add the Product to the former reserv'd Sum, and that Sum will be the triple Square of a mean Diameter; which multiply'd by .7854, and that Product multiply'd by a third Part of the Height, the Product is the solid Content.

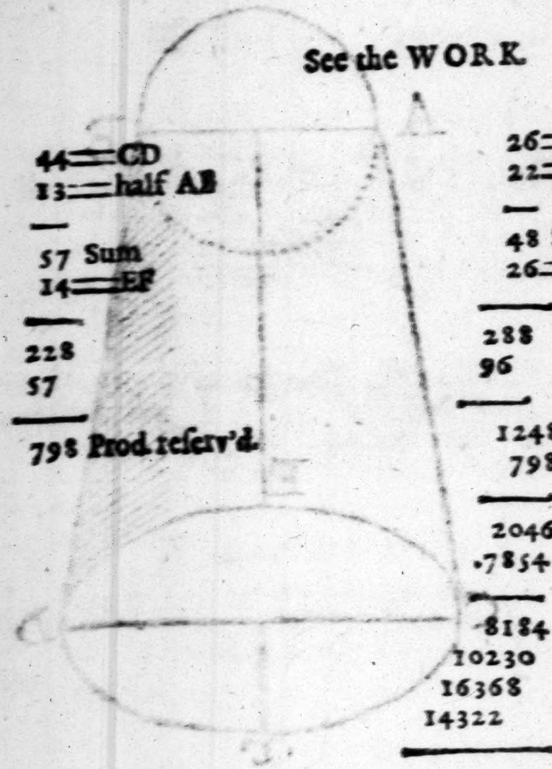
Example



Example. Let AB CD be a Cylindroid, whofe bottom Bafe is an Oval, the tranf-verfe Diameter being 44 Inches, and the conjugate Diameter 14 Inches; and the upper Bafe is a Circle, whofe Diameter is twenty fix Inches, and the Height of the Fruftum is 9 Feet; the Solidity is requir'd.

To 44 (the greater Diameter of the lower Bafe) add 13, half the Diameter of the leffer Bafe) the Sum is 57; which multiply'd by 14, (the conjugate Diameter of the greater Bafe) the Product is 798: which referve. Then to 26 (the Diameter of the leffer Bafe) add 22, (half the tranfverfe Diameter of the greater Bafe) and the Sum is 48; which multiply'd by 26, the Diameter of the leffer Bafe) the Product is 1248; to which add the former referv'd Product, the Sum is 2046; which multiply'd by .7854, and the Product is 1606.9284; which multiply'd by 3, (a third Part of the Height) the Product is 4820.7852; which divided by 144, the Quotient is 33.47 Feet, the folid Content.

See the WORK.



44 = CD
13 = half AB
—
57 Sum
14 = EF
—
228
57
—
798 Prod. reserv'd.

26 = AB
22 = half CD
—
48 Sum.
26 = AB
—
288
96
—
1248
798 Add.
—
2046
.7854
—
8184
10230
16368
14322
—
1606.9284
3
—
144)4820.7852(33.47
500
687
1118
—
110

This Rule being the same with that in the last Section, the Proof of that may serve as a sufficient Proof of this, if what has been before written be well consider'd.

To find the superficial Content.

To the Periphery of the Ellipsis 97.41 add the Periphery of the Circle 81.68, and the Sum is 179.09, the half thereof 89.545, multiply'd by 9, the Product is 805.905; which divided by 12, the Quotient is 67 16 Feet, the Curve-Surface: Then

Then the Area of the Ellipſis is 3.36 Feet, and the Area of the Circle is 3.68 Feet; both which added to the Curve-Surface, the Sum is 74.2 Feet, the whole ſuperficial Content.

§ XI. Of a SPHERE or GLOBE.

A SPHERE, or GLOBE, is a round ſolid Body, every Part of whoſe Surface is equally diſtant from a Point within it, call'd its Center; and it may be conceiv'd to be form'd by the Revolution of a Semicircle round its Diameter To find its Solidity, this is

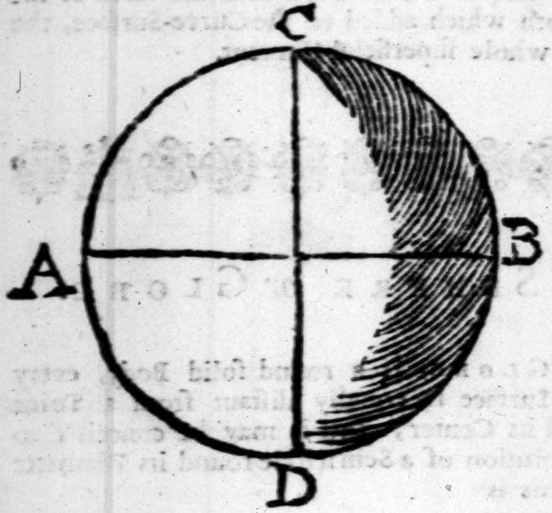
The RULE.

1. Multiply the Axis, or Diameter into the Circumference, the Product is the Superficial Content; which multiply by a ſixth Part of the Axis, the Product is the Solidity.

2. Or thus: As 21 : to 11 : : ſo is the Cube of the Axis to the ſolid Content.

3. Or, As 1 : is to .5236 : : ſo is the Cube of the Axis to the ſolid Content.

Example



Example. Let A B C D be a Globe, whoſe Axis is 20 Inches then the Circumference will be 62.832: Then, by the firſt Rule, multiply the Circumference by the Axis and the Product will be 1256.64, which is the ſuperficial Content in Inches; take a ſixth Part thereof, which is 209.44, (becauſe an exact ſixth Part of 20 cannot be taken) multiply that ſixth Part by 20, (the Axis) and the Product is 4188.8, the Solidity in Inches. Or, if you multiply the ſuperficial Content by the Axis, and take a ſixth Part of the Product, the Anſwer will be the ſame.

Or thus, by the ſecond Rule:

The Cube of the Axis is 8000, which multiply'd by 11, the Product is 88000; which divided by 21, the Quotient is 4190.47, the Solidity.

Or, by the third Rule:

If the Cube of the Axis be multiply'd by .5236, the Product is 4188.3 the Solidity, the ſame as by the firſt Way. If you divide 4188.8 by 1728, the Quotient is 2.424 Feet.

See the WORK.

62.832

62.832
20

6)1256.640 the ſuperficial Content.

209.44 a ſixth Part.
20

4188.80 the Solidity in Inches.

As 21 : 11 : : 8000
11

21)88000(4190.47 the Content

40
190
100
160

13

As 1 : .5236 : : 8000
8000

1728)4188.8000(2.424 Feet, the Solidity.

7328
4160
7040

128

NOTE, If the Axis of a Globe be 1, the Solidity will be .5236; and if the Circumference be 1, the Solidity will be .016887.

By Scale *and* Compaſſes.

Extend the Compaſſes from 1 to 20, (the Axis) that Extent turn'd three times over, from .5236) will at the laſt fall upon) 4188.8, the ſolid Content in Inches: Or, Extend the Compaſſes

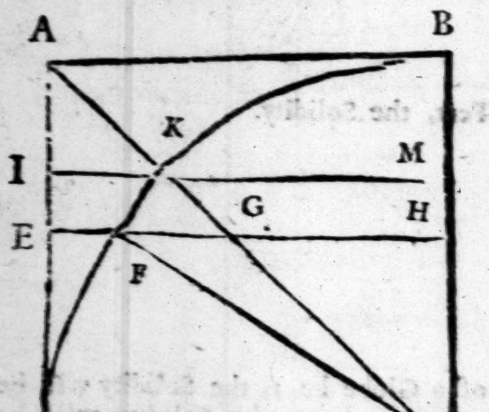
passes from 1728 to 8000, (the Cube of the Axis) that Extent will reach from .5236 to 2.424, the solid Content in Feet.

Extend the Compasses from 1 to 20, (the Axis) that Extent (turn'd twice over from 3.1416) will at last fall upon 1256.64, the superficial Content in Inches: Or, Extend the Compasses from 144 to 400, (the Square of the Axis) that Extent will reach from 3.1416 to 8.72, the superficial Content in Feet.

DEMONSTRATION.

Every Sphere is equal to a Cone whose perpendicular Axis is the Radius of the Sphere, and its Base a Plane, equal to all the Surface of it.

For you may conceive the Sphere to consist of an infinite Number of Cones, whose Bases, taken all together, compose the Surface, and whose Vertexes meet all together in the Center of the Sphere: Hence the Solidity of the Sphere will be gain'd, by multiplying its Surface by $\frac{1}{3}$ of its Radius.



Let the Square ABCD, the Quadrant CBD, and the right-angled Triangle ABD, be suppos'd all three to revolve round the Line BD as an Axis; Then will the Square generate a Cylinder, the Quadrant an Hemisphere, and the Triangle a Cone, all of the same Base and Altitude.

Then the Square of EH $= \square$ FD $= \square$ FH $+ \square$ DH (but DH = GH.) And since Circles are as the Squares of their Diameters, (by *Euclid* 12. 2.) the Circle made by the Revolution of EH must be equal to both the Circles made by the Motions of FH and GH.

If you take the Circle made by the Revolution of FH from both, there will remain the Circle made by the Motion of GH, equal to the Ring describ'd by the Motion of EF. And thus it will always be, where-ever you draw the Line EH or IM, &c.

Therefore the Aggregate, or Sum, of all the Rings made by the Revolution of EF's, must be equal to that of all the Circles made by the Motion of GH's, *i. e.* the Dish-like Solid, form'd by the revolving Rings, will be equal to the Cone, form'd by the the Revolution of the GH's, which are the Elements of the Triangle ABD; that is, the Dish-like Solid, will be as the Cone, $\frac{1}{3}$ of the circumscribing Cylinder, and consequently the Hemisphere must be $\frac{2}{3}$ of it: Wherefore the Sphere is $\frac{2}{3}$ of the circumscribing Cylinder.

Let the Radius of the Sphere be $r = CD$, the Diameter will be $2r$; let the Surface of the Sphere, generated by the revolving Semicircle, be call'd S, and that of the Cylinder, form'd by the Revolution of $2\,AC = 2r =$ Diameter, be call'd ſ. Wherefore, in what was just now prov'd, the Expression for the Solidity of the Sphere in this Notation, will be $\frac{rS}{3}$, and putting c equal to the Circumference of the Base, or for the Periphery of a great Circle of the Sphere, the Curve-Surface of the Cylinder will be $2rc$; also $\frac{rc}{2}$ will be the Area of a great Circle, (by Sect. IX. of Chap. I. *Problem* 1.) and this multiply'd by $2r$, makes rrc, which is the Solidity of the Cylinder, by Sect. V. of this Chapter. Now, since ſ was put equal to $2rc =$ to the Curve-Surface of the Cylinder, $\frac{ſr}{2}$ (by substituting ſ for $2rc$) will be also = to the Solidity of the Cylinder. Now, since the Sphere is $= \frac{2}{3}$ of the Cylinder, $\frac{rS}{3} = \frac{2}{3}\,\frac{ſr}{2}$; that is, $\frac{rS}{3} = \frac{2ſr}{6} = \frac{ſr}{3}$.

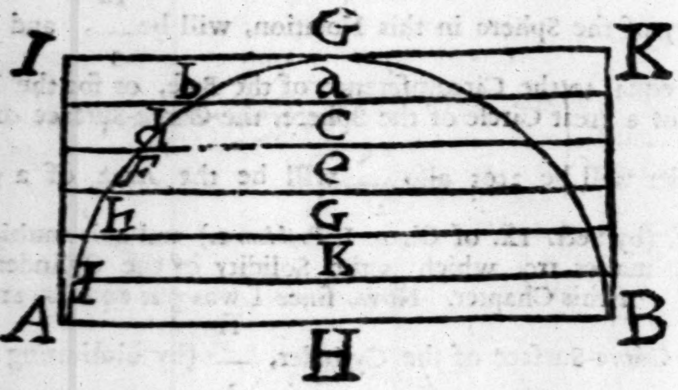
Wherefore $rS = rſ$, that is, dividing by r, $S = ſ$; or the Surface of the Sphere, is equal to the Curve-Surface of the Cylinder, but the Curve Surface of the Cylinder was $2rc$.

Where

Wherefore, to find the Area of the Surface of either Sphere or Cylinder, you must multiply the Diameter ($=2r$) by the Circumference of a great Circle of the Sphere, or by the Periphery of the Base. From this Notation also $\frac{rc}{2}$, the Area of a great Circle of the Sphere, is plainly $\frac{1}{4}$ of $2rc$, the Surface of the Sphere; that is, the Surface of the Sphere is Quadruple of the Area of the greatest Circle of it.

Wherefore, to $2rc$, the Convex-Surface of the Cylinder, add rc, equal to the Area of both its Bases, you will have $3rc$; which shews you, that the Surface of the Cylinder (including its Bases) is the Surface of the Sphere as 3 to 2; or that the Sphere is $\frac{2}{3}$ of the circumscribing Cylinder, in Area as well as Solidity.

¶ Or you may prove the Sphere to be $\frac{2}{3}$ of the Cylinder of the ame Base and Altitude, by LEMMA VI aforegoing thus:



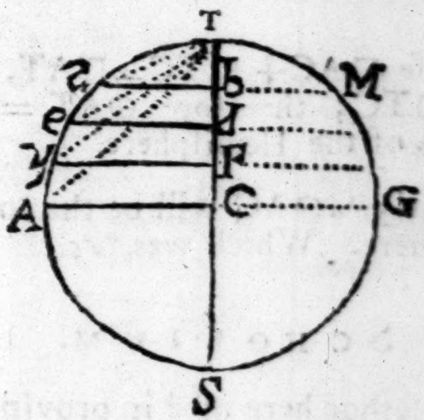
¶Let AGB represent the Hemisphere, and AIKB half the Cylinder; then, if the Semidiameter GH be divided into six equal Parts, and Lines drawn parallel to AB, the Diameter, the Squares of the Semichords, ab, cd, ef, &c. will be a Series of Numbers, whose greatest Term AH is a square Number, the other differing by odd Numbers; that is, AH is 36, kl 35. gh 32, ef 27, cd 20, ab 11: But an infinite Series of such Numbers are in Proportion to an infinite Number of Terms equal to the greatest, as 2 to 3. And because the Hemisphere is compos'd of

of an infinite Number of Circles, whoſe Diameters are the Chords of the Semicircle; and the half Cylinder is compos'd of an infinite Number of Circles, whoſe Diameters are all equal to the Diameter of the Semicircles AB; therefore the Hemiſphere is in Proportion to the half Cylinder; as 2 to 3; and, conſequently, the whole Sphere bears the ſame Proportion to the whole Cylinder.

That the Superficies of every Sphere (or Globe) is equal to four times the Area of its greateſt Circle, is thus prov'd:

The Solidity of the Sphere is conſtituted of an infinite Number of Parallel Circles (as is aforeſaid;) conſequently, the Superficies of the Sphere will be compos'd of the Peripheries of thoſe Circles which conſtitute its Solidity.

NOTE, In the following Demonſtrations, ⊙ ſignifies any Circle in general: And if any two Letters be join'd to it, thus ⊙AB, *&c.* then it denotes the Area of ſuch a Circle as thoſe two Letters repreſent the Radius of.



Let D = TS, the Axis of any Sphere; then, according to the Property of a Circle, ir

will

will be	1	D — Tb × Tb = □ab;
that is,	2	D × Tb — □ Tb = □ ab;
therefore	3	D × Tb = □ aT.
For	4	□ab + □ Tb = □ aT, (*Euc.* 1. 47.)
and	5	D × Td = □ eT.
	6	D × Tf = □ yT.

Hence it is evident that the Series □ aT. □eT, □yT. *&c.* are in the fame Ratio with Tb, Td, Tf, *&c. viz.* are in arithmetical Progreffion: Whence it follows, that the ⊙ aT = to the Sum of all the Circle's Peripheries between T and b.

And ⊙eT = the Sum of all the Circle's Peripheries between T and d, *&c.*

Confequently, that the ⊙ AT = the Sum of all the Circle's Peripheries, included between T and C; that is, ⊙ A T = the Superficies of the Hemifphere.

And becaufe □AC + □TC = □AT, and □ AC is equal to □TC; therefore ⊙AT = 2⊙AC, is the Superficies of the Hemifphere.

Confequently, 4⊙AC will be the Superficies of the whole Sphere. Which was, *&c.*

Scholium.

From the Method here ufed in proving the whole Superficies, it will be eafy to find the Curve-Superficies of any Fruftum, or Part of a Sphere, that is cut off by a right Line, or Plane, *viz.* fuch as the Fruftum aTm in the laft Scheme, whofe Curve-Superficies is ⊙aT, as above. Therefore (becaufe □ ab + □ Tb = □aT) it will be ⊙ab + ⊙Tb = the Curve-Superficies of that Fruftum.

But

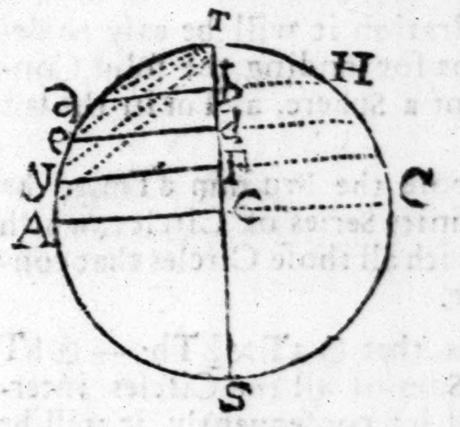
But if the Axis TS, and Height Tb of the Frustum, are given, then it will be TS × Tb = □ aT, as in the third Step above, which gives the Proportion or Theorem following, *viz.*

As the Axis of the Sphere : to the whole Superficies of the Sphere : : so is the Height of any Frustum : to its Curve-Superficies.

To which, if there be added the Area of the Frustum's Base, the Sum will be the whole Superficies of the Frustum.

That the Solidity of every Sphere is two Thirds of its Circumscribing Cylinder, *may be thus prov'd.*

According to the Work above, it appears, that ⊙ab, ⊙ed, ⊙yf, &c. do constitute the Solidity of the Sphere; and that □aT, □eT, □yT, &c. are a Series of Terms in Arithmetical Progression, □ AT being the greatest Term, and TC the Number of Terms; therefore ⊙ AT × $\frac{1}{2}$ TC = the Sum of all the Series, by Lemma 2.



And because □ aT — □ Tb = □ ab. □eT — □Td = □ed. □yT — □Tf = □yf. □ AT — □TC = □ AC, &c. wherein □Tb, □Td, □Tf, &c. are a Series of Squares, whose Roots Tb, Td, Tf, are in arithmetical Progression; □TC being the greatest Term and TC the Number of Terms; therefore ⊙TC × $\frac{1}{3}$ TC = the Sum of all that Series, by Lemma 3.

Con-

Confequently, ⊙ AT $\times \frac{1}{2}$ TC :—⊙TC $\times \frac{1}{3}$ TC= the Sum of all the Series ⊙ab. ⊙ed, ⊙yf, &c. which conftitute the Solidity of the half Sphere ATG. Put D =2TC the Axis of the Sphere; then $\frac{1}{4}$ D= $\frac{1}{2}$ TC, and $\frac{1}{6}$ D = $\frac{1}{3}$ TC. And becaufe □ AT=2□TC, therefore ⊙AT=2⊙TC=1.5708DD; and 1.5708 DD $\times \frac{1}{4}$ D=0.3927 DDD.

Again, ⊙TC $\times \frac{1}{3}$ TC=0.7854 DD $\times \frac{1}{6}$ D=.1309 DDD, then 0.3927 DDD—0.1309 DDD, =0.2618 DDD, the Solidity of the half Sphere.

Confequently, 0.2618DDD $\times$ 2= .5236 DDD, will be the folid Content of the whole Sphere, which is equal to $\frac{2}{3}$ of the Cylinder; the Diameter of whofe Bafe, and alfo its Height, is = D.

For 0.7854 DDD = the Solidity of the Cylinder by Sect. V. But $\frac{2}{3}$ of 0.7854 DDD= 0.5236 DDD, as before.

SCHOLIUM.

From this Demonftration it will be eafy to deduce or raife Theorems for finding the folid Content of any Fruftum of a Sphere, as Tm in the laft Figure.

For we there fuppofe the Fruftum aTm to be conftituted of an infinite Series of Circles, which have the fame Ratio with all thofe Circles that conftitute the half Sphere.

Therefore it follows, that ⊙ aT $\times \frac{1}{2}$ Tb :— ⊙bT $\times \frac{1}{3}$ Tb will be the Sum of all the Circles intercepted between T and b; confequently it will be the Solidity of that Fruftum.

And becaufe □ ab+□Tb=aT; therefore ⊙ab +⊙Tb$\times \frac{1}{2}$Tb : —⊙Tb $\times \frac{1}{3}$ Tb= the Solidity. Let c=ab half the Diameter of the Fruftum's Bafe; h= Tb its Height; and S= the Solidity of the Fruftum. Then ⊙ab=3.416 cc, and ⊙Tr=3.1416

hh

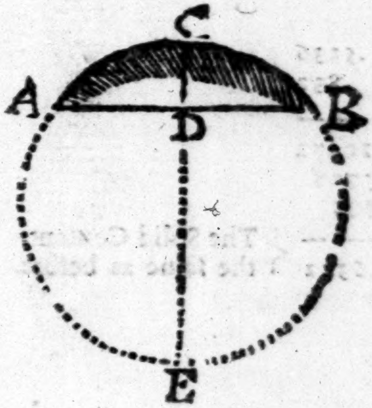
hh; conſequently, $\frac{3.1416cch + 3.1416hhh}{2} - \frac{3.1416hhh}{3} = S$

Which being reduc'd, will become $\overline{3cch + hhh}\ 0.5236 = S$; which is one Theorem for finding the Solidity of the Fruſtum, and may be expreſs'd in Words thus;

If to three Times the Square of the Semidiameter of the Fruſtum's Baſe you add the Square of the Height of the Fruſtum, and multiply the Sum by the Height of the Fruſtum, and that Product multiply'd by .5236, the Product will be the ſolid Content.

But if the Axis of the Sphere, and the Height of the Fruſtum, be given; then put D = the Axis, h = the Height of the Fruſtum, and c as before; it will be $\overline{D - h} \times h = cc$, viz. $Dh - hh = cc$. Then will $3Dhh - 2hhh = 3cch + hhh$; conſequently $\overline{3Dhh - 2hhh} \times 0.5236 = S$, the Fruſtum's Solidity. Which is another Theorem for finding the Solidity of the Fruſtum, and may be expreſs'd in Words thus:

From three times the Axis ſubtract twice the Height of the Fruſtum, and multiply the Remainder by the Square of the Height, and that Product multiply by .5236, this laſt Product will be the Solidity of the Fruſtum.



Example. Let ABCD be the Fruſtum of a Sphere; ſuppoſe AB, (the Diameter of the Fruſtum's Baſe) be 16 Inches, and CD (the Height) 4 Inches; the Solidity is requir'd.

By

By the first Rule.

```
    8
    8
   ——
   64 Square of the Semidiameter AD.
    3
  ———
  192
   16 Add the Square of CD.
  ———
  208
    4 Multiply by CD.
  ———
  832              .5236
                     832
                  ———————
                   10472
                  15708
                 41888
                 ————————
   The Solidity 435.6352
```

By the second Rule thus.

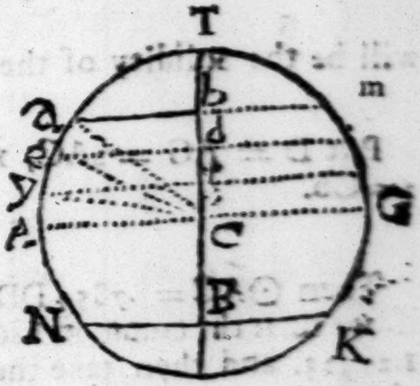
First, by the Rule in Page 109, you will find the Axis of the whole Globe to be 20 Inches.

```
         20 Axis.              .5236
          3                     832
        ———                  ———————
   From  60                    10472
   Subtr. 8 twice CD.         15708
        ———                  41888
   Rem.  52                 ————————    } The Solid Content,
   Mult. 16 Square of CD.   435.6352    } the same as before.
        ———
        312
        52
        ———
   Prod. 832
```

And

And if it be requir'd to find the middle Part, amNK, usually call'd the middle Zone of a Sphere;

Then, because it is suppos'd that am = NK, or (which is all one) that that bC = CB; therefore it is plain, that if twice the Segment, aTm, be taken from the whole Sphere, there will remain the middle Zone amNK.



But because that Work is a little troublesom, I will here shew how to raise a Theorem for the doing it.

First, because $AC = yC = eC = aC = TC$, therefore it will be $\square AC - \square Cf = \square yf$, $\square AC - \square Cd = \square ed$, $\square AC - \square Cb = \square ab$, &c.

Here, because $\square$ AC, $\square$AC, $\square$AC, &c. are a Series of Equals, and Cb the Number of all the Terms; therefore $\square AC \times Cb =$ the Sum of all that Series, per Lemma 1.

And $\square$Cf, $\square$Cd, $\square$Cb, &c. being a Series of Squares, whose Roots are, in arithmetical Progression, beginning at the Center C, *viz.* o. Cf, Cd, Cb, &c. wherein the greatest Term is $\square$Cb, and the Number of Terms is Cb; therefore $\square Cb \times \frac{1}{3} Cb =$ the Sum of all the Series, per Lemma 3.

Consequently, the $\odot AC \times Cb : - \odot Cb \times \frac{1}{3} Cb =$ the Sum of all the Series $\odot$yf, $\odot$ed, $\odot$ab, &c. which do constitute the Solidity of the Half Zone amAG.

 And

And becauſe $\square AC - \square Cb = \square ab$, therefore $\odot AC - \odot ab = \odot Cb$. Conſequently $\odot AC \times Cb: - \odot AC + \odot ab: \times Cb$

$$\frac{\odot AC \times Cb: - \odot AC + \odot ab: \times Cb}{3} = 2\,\odot AC + \odot ab: \times \tfrac{1}{3} Cb$$

will be the Solidity of the half Zone.

Put $D = AG = 2AC$, $x = am$, and $H = bB = 2 = Cb$.

Then $\odot AC = .7854DD$. $\odot ab = .7854xx$. And if we turn the common Factor .7854 into a Diviſor, 1.27323, and then take the Triple of that Diviſor, *viz.* 3.8197, the Reſult of the Precedent Work will produce this following Theorem.

$$\text{Theor.}\left\{\frac{2DD + xx}{3.8197}\right\}: x\,H = \left\{\begin{array}{l}\text{the middle Zone}\\ \text{amNK;}\end{array}\right.$$

Which in Words is thus; To twice the Square of the Axis AG. add the Square of the Diameter of the Fruſtum's Baſe (am) and divide the Sum by 3.8197, then multiply the Quotient by the Height or Thickneſs of the middle Zone, and the Product will be the Solidity of the middle Zone requir'd.

This is ſo plain and eaſy, it needs no Example.

§ XII.

§ XII. *Of a SPHEROID.*

A SPHEROID is a Solid reſembling an Egg. To find the Solid Content thereof, this is

The RULE.

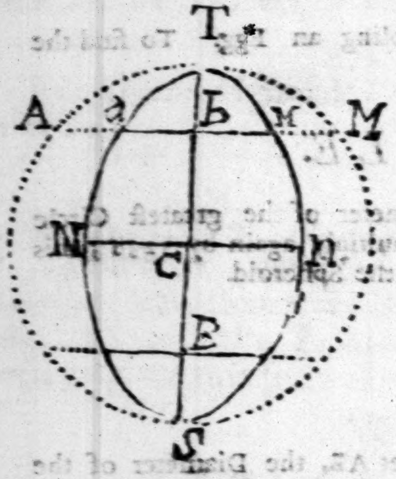
Multiply the Square of the Diameter of the greateſt Circle by the Length, and that Product multiply again by .5236; this laſt Product will be the Solidity of the Spheroid.

Let AB, the Diameter of the greateſt Circle, be 33 Inches, and CD (the Length) 55 Inches; the Solidity is requir'd.

```
  33          59895
  33          .5236
 ----        -------
  99         359370
 99         179685
 ----       119790
 1089      299475
   55      -----------
 ----      31361.0220 the Solidity.
 5445
5445
-----
59895
```

DEMONSTRATION.

Every Spheroid is equal to $\frac{2}{3}$ of a Cylinder, whoſe Baſe is equal to the greateſt Circle of the Spheroid, and its Height equal to the Length of the Spheroid.



Suppoſe the Figure NTnSN in the annex'd Scheme, to repreſent a Spheroid, form'd by the Rotation of the Semi-Ellipſis TNS, about its tranſverſe Axis TS.

Let D = TS, the Length of the Spheroid, and the Axis of its circumſcribing Sphere; and d = Nn the Diameter of the greateſt Circle of the Spheroid.

Then, becauſe □ TC : □ NC :: □ Ab : □ ab, by *Sect.* XV, *Step.* 3, *Page* 119.

Therefore it will be DD : dd :: □ Ab : □ ab.

But the Sum of an infinite Series of ſuch Circles as ⊙ ab (whoſe Diameters are Chords) do conſtitute the Solidity of the Sphere. (By *Sect.* XI.)

And the Sum of an infinite Series of ſuch Circles as ⊙ ab (*viz.* whoſe Diameters are Ordinates of the Ellipſis) do conſtitute the Solidity of the Spheroid.

Therefore DD : dd :: 0.5236DDD : 0.5236 Ddd = the Solidity of the Spheroid. (*Eucl.* 5, 12.)

But 0.5236 Ddd = $\frac{2}{3}$ of the Cylinder, whoſe Diameter is = d, and Height = D. (By *Sect.* V.)

Now, from this Proportion between the Sphere and its inſcrib'd Spheroid, it will be very eaſy to deduce Theorems for finding the ſolid Content, either of the Fruſtum or middle Zone of any Spheroid, having the ſame Height with that of the Sphere; for,

As the Solidity of the whole Sphere : is to the Solidity of the whole Spheroid :: ſo is any Part of the Sphere, to the like Part of the Spheroid.

As for Instance, suppose it was requir'd to find the middle Zone of any Spheroid.

Let D = TS, and d = Nn, as above; and H = bB, x = AM, and c = am.

Then $\left\{ \dfrac{2DD + xx}{3.8197} \times H \right.$ = the middle Zone of the Sphere.

And $0.5236DDD : 0.5236ddD :: \dfrac{2DD + xx}{3.8197} \times H : \dfrac{2ddH}{3.8197} + \dfrac{xxddH}{3.8197DD}$ = the middle Zone of the Spheroid.

Again, $DD : dd :: xx : cc$. Therefore $\dfrac{xxdd}{DD} = cc$.

Consequently, $\dfrac{xxdd}{DD} \times \dfrac{H}{3.8197} = \dfrac{cc}{3.8197} \times H$. Which being taken instead of $\dfrac{xxddH}{3.8197DD}$, there will arise

This following Theorem, $\left\{ \dfrac{2dd + cc}{3.8197} \times H \right.$ = the middle Zone of the Spheroid.

NOTE, That $3.8197 = 1.2732 \times 3$. *See Page* 101.

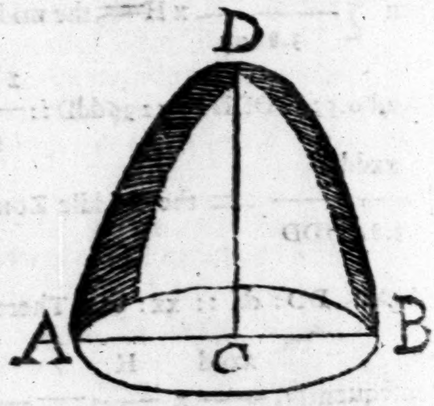
§ XIII. *Of a Parabolick* CONOID.

A PARABOLICK Conoid is something like an half Spheroid, having its Sides somewhat straighter. It is generated by supposing a Semi-Parabola turn'd about its Axis. To find the solid Content thereof, this is

The RULE.

Multiply the Square of the Diameter of its Base by .7854, and multiply that Product by half the Height, that last Product shall be the solid Content.

Let ABCD be a Parabolick Conoid, the Diameter of whose Base is 36 Inches, and its Height CD 33 Inches; the Solidity is requir'd.



36	.7854	1017.8784
36	1296	33
216	47124	30536352
108	70686	30536352
	15708	
1296	7854	2)33589.9872
	1017.8784	16794.9936

1728)16794.99|36(9.71 Feet, the Content.
15552

12429
12096

3339
1728

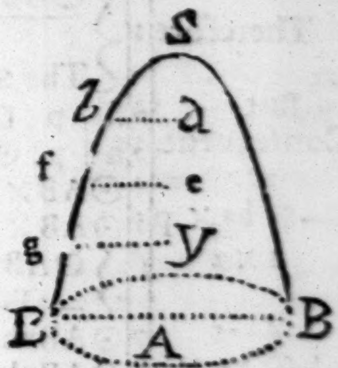
1611

DEMONSTRATION.

The Parabolick Conoid is constituted of an infinite Number of Circles, whose Diameters are the Ordinates of the Parabola. Now, according to the Property of every Parabola, will be $SA : AB :: AB : \frac{\square AB}{SA} = L$, the *Latus Rectum.*

Then

$$\text{Then}\begin{cases} Sa\times L=\square\, ba,\\ Se\times L=\square\, fe,\\ Sy\times L=\square\, gy, \&c.\end{cases}$$



Here $Sa\times L$, $Se\times L$, Sy, $\times L$, &c. are a Series of Terms in Arithmet. Progression. Therefore $\square$ ba, $\square$ fe, $\square$gy, &c. are also a Series of Terms in the same Progression, beginning at the Point S, wherein $\square$ AB is the greatest Term, and SA the Number of all the Terms. Therefore $\square\,AB\times\frac{1}{2}SA$ = the Sum of all the Series. (By Lemma 2.)

Consequently, $\odot\,AB\times\frac{1}{2}SA$ = the Sum of all the Series of $\odot$ ba, $\odot$ fe, $\odot$ gy, &c. which do constitute the Solidity of the Conoid.

Put $D=2AB$, and $H=SA$.

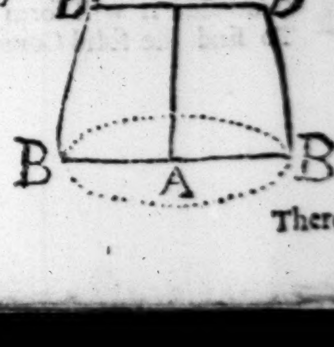
Then $.7854DD\times\frac{1}{2}H=.3927DDH$ will be the solid Content of the Conoid; which is just half the Cylinder, whose Base is $=D$, and the Height $=H$.

This being rightly understood, it will be easy to raise a Theorem for finding the lower Frustum of any Parabolick Conoid.

For, supposing $h=1A$, the Height of the Frustum, and $p=Sa$, the Height of the Part bSb cut off; then $h+p=SA$, the Height of the whole Conoid.

Consequently $\dfrac{\odot AB\times h+\odot AB\times p}{2}$ = the Solidity of the whole Conoid.

And $\dfrac{\odot\, ba\times p}{2}$ = the Solidity of the Part cut off.



There-

Therefore	1	$\frac{\odot AB \times h + \odot AB \times p - \odot ba \times p}{2}$, The Solidity of the Frustum.
But	2	$h + p : \square AB :: p : \square ba$.
Consequent.	3	$h + p : \odot AB :: p : \odot ba$.
3 ∵	4	$\odot AB \times p = \odot ba \times h + \odot ba \times p$.
$4 - \odot ba \times p$	5	$\odot AB \times p : - \odot ba \times p = \odot ba \times h$.
1×2	6	$\odot AB \times h : + \odot AB \times p : - \odot ba \times p = 2F$.
$6 - 5$	7	$\odot AB x h = 2F : - \odot ba x h$.
$7 + \odot ba \times h$	8	$\odot AB x h : + \odot ba x h = 2F$.
$8 \div 2$	9	$\frac{\odot AB + \odot ba}{2} : x\ h = F$, the Frustum's Solidity.

Let $D = 2AB$, as before, and $d = 2ba$, the Diameter of the Part cut off; then we shall have this following Theorem.

$0.3927\ DD + 0.3927\ dd : x\ h =$ the Solidity of the Frustum requir'd: Which in Words is thus;

Multiply the Sum of the Squares of the greater and lesser Diameters, by .3927, and that Product by the Height of the Frustum, the last Product shall be the solid Content.

§ XIV. Of a Parabolick SPINDLE.

If an acute Parabola be suppos'd to be mov'd about its greatest Ordinate, it will form a Solid call'd a Parabolick Spindle. To find the solid Content, this is

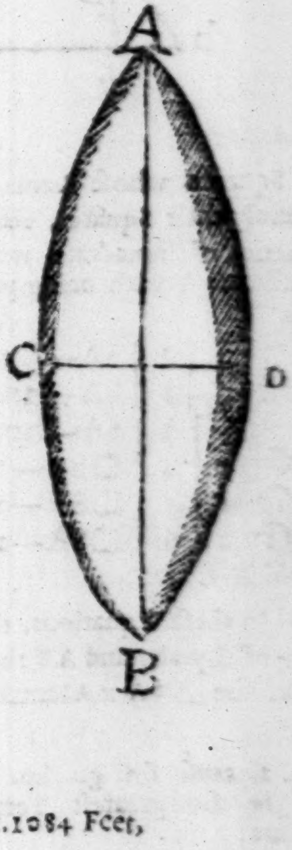
The

The RULE.

Multiply the Square of the Diameter of its greatest Circle by .41888, (being $\frac{8}{15}$ of .7854) and that Product by its Length; that last Product is the solid Content.

Let ABCD be a Parabolick Spindle, whose greatest Diameter CD is 36 Inches, and its Length AB 99 Inches; the Solidity is requir'd.

```
  36 = CD.        .41888
  36                1296
 ----            -------
  216             251328
 108             376992
 ----            83776
 1296 Square     4188
                 ---------
                 542.86848
                        99
                 ---------
               488581632
              488581632
              ---------------
1728)53743.97952(31.10114
     ----
      1903
       1759
        3179
        14515
          0612
          ----
```



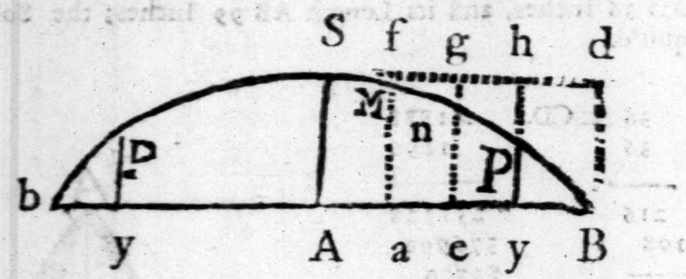
The solid Content is 31.1084 Feet,

DE-

DEMONSTRATION.

A Parabolick Spindle is conſtituted of an infinite Series of Circles, whoſe Diameters are all Parallel to the Axis of the Parabola, as ⊙ma. ⊖ne, ⊙py &c.

Let us ſuppoſe the Line Sp parallel to AB, *&c.* Then it hath already been prov'd, that the Lines fm, gn, hp, *&c.* are a Series



of Squares, whoſe Roots are in arithmetical Progreſſion; conſequently their Squares, *viz.* □fm, □gn, □hp, *&c.* will be a Series of Biquadrats, whoſe Roots will be in arithmetical Progreſſion: Which being premis'd; we may proceed thus:

First { 1 | SA—fm = ma.
2 | SA—gn = ne.
3 | SA—hp = py.

1 ⊙ 2 | 4 | □SA—2SAxfm+□fm=□ma,
2 ⊙ 2 | 5 | □SA—2SAxgn+□gn=□ne.
3 ⊙ 2 | 6 | □SA—2SAxhp+□hp=□py, *&c.*

1. In theſe Equations, the □SA, □SA □SA, being a Series of Equals, and AB the Number of all the Terms; therefore it will be □SA x AB = the Sum of all the Series, by Lemma I.

2. Becauſe fm, gn, hp, *&c.* are a Series of Squares, wherein SA is the greateſt Term, and AB the Number of all the Terms:

Therefore $\frac{2SA \times SA \times AB}{3} = \frac{2\square SA \times AB}{3}$ will be the Sum of all the Series by Lemma III.

CHAP.

3. And the $\square fm$, $\square gn$, $\square hp$. &c. will be a Series of Terms in the Ratio of Biquadrats, as above, $\square SA$ being the greateſt Term, and AB the Number of all the Terms. Therefore it will be $\frac{\square SA \times AB}{5}$ = the Sum of all the Series, by Lemma V.

Whence it follows, that $\square SA \times AB - \frac{2\square SA \times AB}{3} + \frac{\square SA \times AB}{5}$ = the Sum of all the Series of $\square ma$, $\square ne$, $\square py$, &c.

That is, $\frac{8\square SA \times AB}{15}$ = the Sum of all the Series, $\square ma$, $\square ne$, $\square py$, &c. Conſequently $\frac{8\odot SA \times AB}{15}$ = the Sum of all the Series of Circles, $\odot ma$, $\odot ne$, $\odot py$, &c. which conſtitute the Solidity of half the Spindle, *viz.* of SAB.

Therefore putting D=2SA, and H=2AB, it will be 0.41888DDH = the Solidity of the whole Parabolick Spindle bSB, being $\frac{8}{15}$ of 0.7854DDH, the Solidity of its circumſcribing Cylinder.

From hence we may alſo raiſe a Theorem for finding the Fruſtum, SApy, of the laſt Figure.

For $\odot SA$ being the greateſt Term, $\odot py$ the leaſt Term, and Ay the Number of all the Terms or Circles included between A and y.

There-

Therefore	1	$\square SA - \frac{2SA \times hp}{3} + \frac{\square hp}{5} : \times Ay = z$ the Sum of all the Series, $\square SA$, $\square ma$, $\square en$, $\square py$.
1×3	2	$3\square SA - 3SA \times hp + \frac{3\square hp}{5} : \times Ay = 3z$.
$2 \div Ay$	3	$3\square SA - 2SA \times hp + \frac{3\square hp}{5} = \frac{3z}{Ay}$
But	4	$\square SA - 2SA \times hp = \square py - \square hp$, *per Step.* 6.
$3 - 4$	5	$2\square SA + \frac{3\square hp}{5} = \frac{3z}{Ay} - \square py + \square hp$.
$5 \pm$ &c.	6	$2\square SA + \square py = \frac{2}{5}\square hp = \frac{3z}{Ay}$

Confeq. 7 $2\odot SA + \odot py - \frac{2}{5}\odot hp : \times \frac{1}{3} Ay = z$, the Sum of all the Series of $\odot SA$, $\odot ma$, $\odot ne$, $\odot py$, which do conftitute the Solidity of the Fruftum SApy. Therefore, putting $D = 2SA$, as before, $C = 2py$, $x = 2hp$, and $H = Ay$, it will be $1.5708 DD + .7854CC - .31416xx : \times \frac{1}{3} H =$ the Fruftum SApy. And if we make $L = 2H$, then $1.5708DD + .7854CC - .31416xx : \times \frac{1}{3} L =$ the Double of that Fruftum, being the middle Zone. Which in Words is thus:

Multiply the Square of the greateft Diameter by 1.5708, and multiply the Square of the leffer Diameter by .7854, and multiply the Square of the Difference of the Diameters by .31416; from the Sum of the two former Products fubtract the latter Product, and multiply the Remainder by one third Part of the Length, and that Product will be the Solidity of the middle Zone requir'd.

CHAP.

CHAP. III.

Of the Measuring of the Works of the several Artificers relating to Building; and what Methods and Customs are observ'd therein.

§ I. *Of* CARPENTERS *Work.*

THE Carpenters Works, which are measurable, are, Flooring, Partitioning, and Roofing; all which are measur'd by the Square of 10 Feet long, and 10 Feet broad; so that one Square contains 100 square Feet.

1. *Of* Flooring.

If a Floor be 57 Feet 3 Inches long, and 28 Feet 6 Inches broad, how many Squares of Flooring are there in that Room?

Multiply 57 Feet 3 Inches by 28 Feet, 6 Inches, and the Product is 1631 Feet, *&c.* which divide by 100; (this is done by cutting off from the Product two Figures towards the right Hand with a Dash of the Pen) and the remaining Figures are the Quotient, and the Figures cut off are Feet. Thus, 1631 divided by 100, by cutting off 31 from the right Hand thereof, the Quotient is 16 Squares, and 31 cut off, is 31 Feet.

See

See the Work, both by Decimals, and also by Feet and Inches.

		F.	I.	
57.25		57	3	
28.5		28	6	
28625				
45800		456		
11450		114		
		28	7	6
16|31.625		7	0	0
		16|31	7	6

Facit 16 Squares and 31 Feet.

NOTE, That .5 is the Decimal for half of any Thing, .25 is the Decimal for a Quarter, and .125 is the Decimal for half a quarter; so, in the last Example, .25 is the Decimal of 3 Inches, because 3 Inches is a quarter of a Foot; and .5 is the Decimal of 6 Inches, because 6 Inches is half a Foot.

Example 2. Let a Floor be 53 Feet 6 Inches long and 47 Feet 9 Inches broad, how many Squares are contain'd in that Floor?

47.75		F.	I.	
53.5		53	6	
		47	9	
23875				
14325		371		
23875		212		
		26	9	
25|54.625		13	4	6
		23	6	
		25|54	7	6

Facit 25 Squares and 54 Feet.

By Scale *and* Compasses.

In the first Example, extend the Compasses from 1 to 28.5, that Extent will reach from 57.25 to 16 Squares and near a third Part.

In the second Example, extend the Compasses from 1 to 47.75, that Extent will reach from 53.5 to 25 Squares and above an half.

2. Of

2. *Of* Partitioning.

Example 1. If a Partition between Rooms be in Length 82 Feet 6 Inches, and in Height, 12 Feet 3 Inches, how many Squares are contain'd therein?

The Length and Breadth being multiply'd together, the Product is 1010.625; which divide by 100, (as before is shew'd) and the Answer is 10 Squares 10 Feet; the Inches or Parts in these Cases, are of no Value.

		F.	I.	
12.25		82	6	
82.5		12	3	
6125		990	0	
2450		20	7	6
9800				
10|10.625		10|10	7	6

Facit 10 Squares 10 Feet.

Example. 2. If a Partition between Rooms be in Length 91 Feet 9 Inches, and in Breadth 11 Feet 3 Inches, how many Squares are contain'd therein?

The Length and Breadth being multiply'd together the Product is 1032 Feet; which divided by 100, the Answer will be 10 Squares and 32 Feet.

		F.	I.	
91.75		91	9	
11.25		11	3	
45875				
18350		1009	3	
9175		22	11	3
9175				
10|32.1875		10|32	2	3

3. *Of* Roofing.

It is a Rule amongft Workmen, that the Flat of any Houfe, and half the Flat thereof, taken within the Walls, is equal to the Meafure of the Roof of the fame Houfe; but this is when the Roof is true pitch'd. For if the Roof be more flat or fteep than the true Pitch, it will meafure to more or lefs accordingly.

Example 1. If a Houfe within the Walls be 44 Feet 6 Inches long, and 18 Feet 3 Inches broad, how many Squares of Roofing will cover that Houfe?

Multiply the Length and Breadth together, and the Product is 812 Feet, the Flat; the half thereof is 406 Feet, which added to the Flat, the Sum is 1218 Feet; which divided by 100, the Anfwer is 12 Squares and 18 Feet.

18.25	F. I.
44.5	44 6
	18 3
9125	
7300	352
7300	44
	11 1 6
Flat 812.125	9 0 0
half 406	
	the Flat 812 . 1 6
12|18	the half 406
	Sum 12|18

Facit 12 Squares 18 Feet.

By Scale *and* Compaffes.

In the firft Example of Partitioning, extend the Compaffes from 1 to 12.25, that Extent will reach from to 92.5 to 10 Squares and one Tenth.

In the fecond Example, extend the Compaffes from 1 to 11.25, that Extent will reach from 91.75 to 10 Squares, and a little lefs than a third Part.

In

In the Example of Roofing, extend the Compasses from 1 to 18.25, that Extent will reach from 44.5 to 812 the Flat; to which add the half thereof, and the Sum is 12.18, which is 12 Squares 18 Feet, as above.

There are other Works about a Building done by the Carpenter, that are measur'd by the Foot, running Measure, that is, by the Number of Feet in Length only; as Cornices, Doors and Cases, Window-Frames. Guttering, Lintels, Sommers, Skirt-Boards, *&c.*

NOTE 1. In measuring of Flooring, after you have measur'd the whole Floor, you must deduct out of it the Well-Holes for the Stairs and Chimneys; and in Partitioning, for the Doors Windows, *&c.* except (by Agreement) they are to be included.

NOTE 2. In measuring of Roofing, seldom any Deductions are made for the Holes for the Chimney-Shafts, the Vacancies for Lutheren-Lights and Sky-Lights; for they are more Trouble to the Workman, than the Stuff which would cover them is worth.

§ *Of* BRICKLAYERS *Work.*

THE Principal is Tiling, Walling, and Chimney-Work.

I. *Of* Tiling.

Tiling is measur'd by the Square of 10 Feet, as Flooring, Partitioning, and Roofing were in the Carpenter's Work; so that between the Roofing and Tiling, the Difference will not be much, yet the Tiling will be the most; for the Bricklayers sometimes will require to have double Measure for Hyps and Vallies. When Gutters are allow'd double Measure, the Way is to measure the Length along the Ridge-Tile, and by that Means the Measure of the Gutters becomes double; it is usual also to

allow double Meafure at the Eaves, fo much as the Projecture is over the Plate, which is commonly about 18 or 20 Inches.

Example 1. There is a Roof cover'd with Tiles, whofe Depth on both Sides (with the ufual Allowance at the Eves, is 37 Feet 3 Inches, and the Length 45 Feet; I demand how many Squares of Tiling are contain'd therein?

F.	I.		
37	3		37.25
45	0		45
185			18625
148			14900
11	3		16|76.25
16|76	3		

Anfwer, 16 Squares 76 Feet.

Example 2. There is a Roof cover'd with Tiles, whofe Depth, on both Sides, with the Allowance at the Eaves) is 35 Feet 9 Inches, and the Length 43 Feet 6 Inches; I demand how many Squares of Tiling are in the Roof?

F.	I.			
43	6			35.75
35	9			43.5
215				17875
129				10725
21	9			14300
10	10	6		15|55.125
17	6			
15|15	1	6		

Here the Length and Depth being multiply'd together, the Product is 1555 Feet; which divided by 100, (as before is taught) the Anfwer is 15 Squares and 55 Feet.

By

By Scale *and* Compasses.

In the first Example extend the Compasses from 1 to 37.25, that Extent will reach from 45 to 16 Squares, and a little above three Quarters of a Square.

In the second Example, extend the Compasses from 1 to 35.75, that Extent will reach from 43.5 to 15 Squares and 55 Feet, that is a little above a half Square.

2. Of Walling.

Bricklayers commonly measure their Work by the Rod-square of 16 Feet and a half; so that one Rod in Length and one in Breadth, contain 272.25 square Feet; for 16.5, multiply'd in it self, produces 272.25 square Feet. But in some Places the Custom is to allow 18 Feet to the Rod; that is, 324 square Feet. And in some Places the usual Way is, to measure by the Rod of 21 Feet long and 3 Feet high; that is, 63 square Feet; and here they never regard the Thickness of the Wall, but the usual Way is to moderate the Price according to the Thickness.

But in *Ireland*, they Measure by a Perch of 21 Feet in Length, and one Foot in Breadth without any regard to thickness in the Measure.

When you measure a Piece of Brick-work, the first Thing is to enquire by which of those Ways it must be measur'd; then having multiply'd the Length and Breadth in Feet together, divide the Product by the proper Divisor, either for Rods or Roods, and the Quotient is square Rods accordingly.

But in *England* commonly Brick-Walls, that are measur'd by the Rod, are to be reduc'd to a Standard Thickness, viz. of a Brick and a half thick, (if it be not agreed on to the contrary;) and to reduce a Wall to Standard Thickness, this is

The RULE.

Multiply the Number of superficial Feet that are found to be contain'd in any Wall, by the Number of Half-Bricks which that Wall is in Thickness; one third Part of that Product shall be the Content thereof in Feet, reduc'd to the Standard Thickness of one Brick and a half.

Example 1. If a Wall be 72 Feet 6 Inches long, and 19 Feet 3 Inches high, and 5 Bricks and a half thick, how many Rods of Brick-work are contain'd therein when reduc'd to the Standard?

19.25 Height.
72.5 Length

9625
3850
13475

1395.625
11

3)15351.875

272.25) 5117.291(18 Rods.

239479

68.06)21679(3 quarters of a rod.

12.61

Anfwer, 18 Rods, 3 Quarters, 12 Feet.

F.	I.	
72	6	
19	3	
648		
72		
18	1	6
9	6	0
1395	7	6
11		

3)15345

272)5115(18 Rods.

2395

68)219(3 Quarters of a Rod

15

NOTE

NOTE, That 68.06 is one fourth Part of 272.25.

NOTE, also, That in reducing of Feet into Rods, they usually reject the odd Parts, and divide only by 272, as is done in the second Way of the last Example; so the Answer, by that second Way, is 18 Rods, 3 Quarters, and 15 Feet, more by about 2 $\frac{1}{2}$ Feet than by the first Way, where it is done decimally; a Thing very insignificant.

Example 2. If a Wall be 245 Feet 9 Inches long, and 16 Feet 6 Inches high, and two Bricks and a half thick, I demand how many Rods of Brick-work are contain'd therein, when reduc'd to Standard-Thickness?

```
       245.75
         16.5
     --------
       122875
      147450
     24575
     --------
     4054.875
            5
     --------
   3)20270
   ---------
 272)6756(24 rods.
   ---------
      1316
     ------
   68)228(3 Quarters of a Rod.
     ----
      24
```

Answer, 24 Rods, 3 Quarters, 24 Feet.

F.	I.	
245	9	
16	6	
1470		
245		
122	10	6
12	0	0
4054	10	6 Anſwer in Feet.

Before I ſhew how to work the two laſt Examples by Scale and Compaſſes, I will ſhew how to find proper Diviſors to facilitate the Operation, becauſe it would be too intricate and tedious to perform by Scale and Compaſſes, according to the Rule above taught.

To find the proper Diviſors.

Divide 3 (the Number of half Bricks in 1 and $\frac{1}{2}$) by the Number of half Bricks in the Thickneſs, the Quotient will be a Diviſor to bring the Anſwer in Feet. But if you would have a Diviſor to give the Anſwer in Rods at once, then multiply 272.25 by the Diviſor found for Feet, and the Product will be a Diviſor which will give the Anſwer in Rods.

Example. Let it be requir'd to find a Diviſor proper to reduce a Wall of three Bricks thick.

Divide 3 by 6, (the half Bricks in the Thickneſs) and the Quotient is .5, which is a Diviſor that will give the Anſwer in Feet. Then multiply 272.25 by .5, and the Product is 136.125, the Diviſor which will give the Anſwer in Rods; that is, as 136.125 is to the the Length of the Wall, ſo is the Height to the Content in Rods. Or, as .5 is to the Length, ſo is the Height to the Content in Feet.

After the ſame Manner you may find Diviſors for any other Thickneſs, which you will find to be as expreſs'd in the following little Table.

The

The Thickneſs of the Wall.	Diviſors for the Anſwer in Feet.	Diviſors for bringing the Anſwer in Rods.
1 Brick thick	1.5	408.375
1 & half Brick thick	1.	272.25
2 Bricks thick	.75	204.1875
2 &half Bricks thick	.6	163.35
3 Bricks thick	.5	136.125
3 &half Bricks thick	.4285	116.678
4 Bricks thick.	.375	102.0937

Let the ſecond Example, aforegoing, be wrought by Scale and Compaſſes, where the Length is 245.75, the Height 16.5, and the Thickneſs is 2 and half Bricks.

Extend the Compaſſes from 163.35 (the tabular Number againſt 2 and half Bricks) to 245.75, that Extent will reach from 16.5 to 24 Rods and 8 Tenths.

Again, if the Length be 75 Feet 6 Inches, and the Height 18 Feet 9 Inches, at 3 and half Bricks thick, how many Rods are contain'd therein?

Extend the Compaſſes from 116.678 (the tabular Number) to 18.75, that Extent will reach from 75.5 to 12.13; that is, 12 Rod and a little above half a Quarter.

In *Britain* it will be very proper and commodious, for ſuch as have frequent Occaſion to meaſure Brick-work, to have in the Line of Numbers little Braſs Center-Pins at each of the Numbers in the third Column of the above little Table, with a Figure to denote the Thickneſs of the Wall.

If a Wall be 104 Feet 9 Inches long, and 17 Feet 3 Inches high, how many Rods are contain'd therein?

```
   104.75
     17.25
   ------
    52375
   20950
  73325
 10475
 -------
63)1806.9375(28
   125
   ---
    546
    504
    ---
     42
```

```
  F.  I.
 104  9
  17  3
 ------
  728
 104
   26  2  3
   12  9  0
 ---------
 1806 11  3
```

Anſwer, 28 Roods, 42 Feet.

NOTE. That ſuch as dig Cellars, do many times do them by the Floor, 18 Feet ſquare, and a Foot deep, being a Floor of Earth; that is 324 ſolid Feet.

3. Of Chimneys.

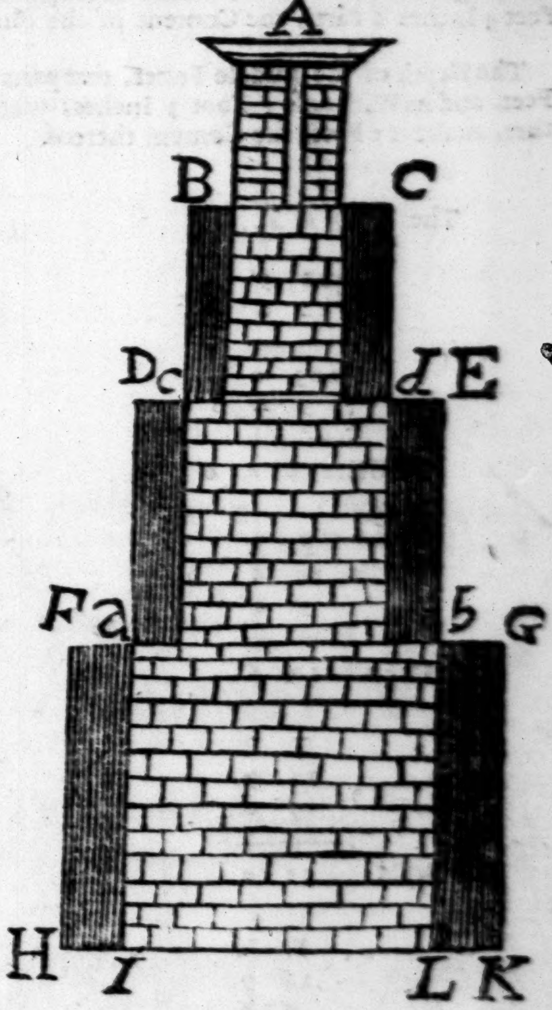
If you are to meaſure a Chimney ſtanding alone by it ſelf, without any Party-Wall being adjoin'd, then girt it about for the Length, and the Height of the Story is the Breadth; the Thickneſs muſt be the ſame as the Jaums are of, provided that the Chimney be wrought upright from the Mantle-tree to the Cieling, not deducting any thing for the Vacancy between the Floor (or Hearth) and the Mantle-tree, becauſe of the Gatherings of the Breaſt and Wings, to make Room for the Hearth in the next Story.

If the Chimney-Back be a Party-wall, and the Wall be meaſur'd by it ſelf, then you muſt meaſure the Depth of the two Jaums, and the Length of the Breaſt for a Length, and the Height of the Story the Breadth, at the ſame Thickneſs your Jaums were of.

When you meaſure Chimney-Shafts, girt them with a Line round about the leaſt Place of them, for the Length, and the Height ſhall be your Breadth; And if they be Four-Inch Work, then you muſt ſet down your Thickneſs at one Brick-work; but if they be wrought 9 Inches Thick, (as ſometimes they are, when they ſtand high and alone above the Roof) then you muſt account your Thickneſs one and half Brick, in conſideration of Wyths and Pargetting, and Trouble in Scaffolding.

It is customary, in most Places, to allow double Measure for Chimneys.

Example. Suppose this Figure ABCDEFGHK, to be a Chimney that hath a double Tunnel towards the Top, and a double Shaft, and is to be measur'd according to double Measure.



First, I begin with the Breast-Wall IL, and the two Angles LK and HI, which together are 18 Feet 9 Inches; then take the Height of the Square HF, 12 Feet 6 Inches, which multiply'd together, produce 234 Feet 4 Inches 6 Parts, for the Content of the Figure FGHK.

For the Square DaEb, the Length of the Breast-Wall and two Angles, is 14 Feet 6 Inches, and the Height Da 9 Feet; which multiply'd together, make 130 Feet 6 Inches, for the Content of the Square DaEb.

Then the Height of the next Square 7 Feet, and the Length of the Breast-Wall and two Angles is 10 Feet 3 Inches; which multiply'd together, produceth 71 Feet 9 Inches, for the Content of the Square BcCd.

The

The Compass of the Chimney-Shafts is 13 Feet 9 Inches, and the Height 6 Feet 6 Inches; which multiply'd together, make 89 Feet 4 Inches 6 Parts, the Content of the Shafts.

The Depth of the middle Fetter, that parts the Funnels, is 12 Feet, and its Wideness 1 Foot 3 Inches; which multiply'd together, make 15 Feet, the Content thereof.

The WORK

	F.	I.			
	18	9			18.75
	12	6			12.5
	225	0			9375
	9	4	6		3750
					1875
FGHK	234	4	6	FGHK	234.375

	F.	I.		
	14	6		14.5
	9	0		9
DaEb	130	6	DaEb	130.5

	F.	I.		
	10	3		10.25
	7	0		7
BcCd	71	9	BcCd	71.75

	F.	I.			
	13	9			13.75
	6	6			6.5
	82	6			6875
	6	10	6		8250
The Shaft	89	4	6	The Shaft	89.375

Feet

	Feet	Inches
	1	3
	12	0
The Fetter	15	0

1.25
12
The Fetter 15.00

272)1082(3 Rods

. 68) 266(3 Quarters.

Rem. 62 Feet.

	F.	I.	P.
FGHK	234	4	6
DaEb	130	6	0
BcCd	71	9	0
The Shaft	82	4	6
The Fetter	15	0	0
The Sum	541	0	0
The Double	1082	0	0

Having added the five Products together, and doubled the Sum, that double Sum is the Content of the Chimney in Feet, according to double or customary Measure; which Feet must be reduc'd to Rods, as was shew'd before.

So the Feet in the foregoing Example being reduc'd to Rods, (the Thickness being suppos'd 1 and half Bricks) it makes 3 Rods 3 Quarters and 62 Feet; that is, 4 Rods wanting 6 Feet.

This is all the Measure that can be allow'd, when the Chimney stands in a Gavel, or Side-Wall; in which Case the Back of the Chimney (here not measur'd) is accounted as Part of the Gavel; but if the Chimneys stand by themselves, as all Stacks of Chimneys in great Buildings do, which, in such Case, is all Chimney-Work, and therefore ought to be measur'd double on all Sides.

§ III. *Of* PLAISTERERS *Work*.

THE Plaisterers Works are principally of two Kinds, namely. 1. Works lath'd or plaister'd, which they call Cieling. 2. Works render'd; which is of two Kinds, *viz.* upon Brick-Walls, or between Quarters, in the Partitions between Rooms; all which are measur'd by the Yard-square, or Square of 3 Feet, which is 9 Feet.

1. *Of*

1. *Of Cieling.*

If a Cieling be 59 Feet 9 Inches long, and 24 Feet 6 Inches broad, how many Yards doth that Cieling contain.

Multiply 59 Feet 9 Inches by 24 Feet 6 Inches, and the Product is 1463 Feet 10 Inches 6 Parts; which divided by 9, the Quotient is 162 Yards 5 Feet.

F.	I.				59.75
59	9				24.5
24	6				
					29875
236					23900
118					11950
29	10	6			
18	0	0			9)1463.875
1463	10	6		Anſwer	162 5

By Scale *and* Compaſſes.

Extend the Compaſſes from 9 to 59 Feet 9 Inches, that Extent will reach from 24 Feet 6 Inches to 162.5 Yards.

2. *Of Rendering.*

Example. If the Partitions between Rooms be 141 Feet 6 Inches about, and 11 Feet 3 Inches high, how many Yards are in thoſe Partitions?

Multiply 141 Feet 6 Inches by 11 Feet 3 Inches, and the Product is 1591 Feet 10 Inches 6 Parts; which divided by 9, gives 176 Yards 7 Feet, the Anſwer.

	141	6	
	11	3	
	1556	6	
	35	4	6
9)	1591	10	6
	Y.	F.	
Answer	176	7	

141.5
11.25
7075
2830
1415
1415
9)1591.875
176.87

Answer, 176.87 Yards.

Extend the Compasses from 9 to 141.5, that Extent will reach from 11.25 to 176.87 Yards.

NOTE 1. If there be any Doors, Windows, or the like, in your Partitioning, you must make Deductions for them.

NOTE 2. When you measure Rendering upon Brick-Walls you are to make no Deductions; but when you measure Rendering between Quarters, you may very well deduct one fifth Part for the Quarters, Braces, and Interstices.

NOTE 3. That Whiting and Colouring are both measur'd by the Yard, as Cieling and Rendering were; and as in Rendering between Quarters, you deduct one fifth Part, so in Whiting and Colouring you must add one fourth, or one fifth Part at least.

§ VI. *Of* JOYNERS *Work.*

JOYNERS do measure their Work by the Yard Square; but in taking their Dimensions, they differ from some others; for they have a Custom, and say, *We ought to measure where our Plane touches*: wherefore, in taking the Height of any Room, where there is a Cornish about, and Swelling Pannels and Mouldings,

dings, they, with a String, begin at the Top, and girt over all the Mouldings; which will make the Room to meaſure much higher than it is: Then, for meaſuring about the Room, they only take it as it is upon the Floor.

Example 1. If a Room of Wainſcot (being girt downwards over the Mouldings) be 15 Feet 9 Inches high, and 126 Feet 3 Inches in Compaſs, how many Yards doth that Room contain?

Multiply the Compaſs by the Height, and the Product is 1988 Feet 5 Inches 3 Parts; which divided by 9, gives 220 Yards and 8 Feet, the Anſwer.

F.	I.			
126	3			126.25
15	9			15.75
630				63125
126				88375
63	1	6		63125
31	6	9		12625
3	9	0		9)1988.4375
9)1988	5	3		220 8
Anſwer 220	8			

Facit 220 Yards 8 Feet.

Example 2. If a Room of Wainſcot be 16 Feet 3 Inches high, (being girt over the Mouldings) and the Compaſs of the Room 137 Feet 6 Inches, how many Yards are contain'd therein?

Multiply 137 Feet 6 Inches by 16 Feet 3 Inches, and the Product is 2234 Feet 4 Inches 6 Parts; which divided by 9, the Quotient is 248 Yards and 2 Feet.

F.	I.	
137	6	
16	3	
830		
137		
34	4	6
9)2234	4	6
248	2	0

137.5
16.25
6875
2750
8250
1375
9)2234.375
248 2

Facit 248 Yards 2 Feet.

By Scale *and* Compasses.

For the first Example, Extend the Compasses from 9 to 126.25, that Extent will reach from 15.75 to 220.8 Yards.

For the second Example, extend the Compasses from 9 to 137.5, that Extent will reach from 16.25 to 248 Yards and about a Quarter.

In Joyners Work there is another Thing to be observ'd, that is, in the measuring of Doors, Window-Shutters, and all such Work as is wrought on both Sides, they are paid for Work and half Work; so that in measuring all such Work, you must first find the Content, as before, and take half that Content and add to it; so shall the Sum be the Content at Work and half.

Example. If the Window-Shutters about a Room be 69 Feet 9 Inches broad, and 6 Feet 3 Inches high, how many Yards are contain'd therein at Work and half?

Multiply 69 Feet 9 Inches by 6 Feet 3 Inches, and the Product is 435 Feet 11 Inches 3 Parts; the half whereof is 217 Feet 11 Inches 7 Parts; which added together, the Sum is 653 Feet 10 Inches 10 Parts; which divided by 9, the Quotient is 72 Yards 5 Feet, the Content at Work and half

F.	I.			
69	9			69.75
6	3			6.25
418	6			34875
17	5	3		13950
				41850
435	11	3		435.9375
217	11	7		217.9687
9)653	10	10		653.9062
72	5			

Facit 72 Yards 5 Feet.

By Scale *and* Compaſſes.

Extend the Compaſſes from 9 to 69.75, that Extent will reach from 6.25 to 48.4 Yards; the Half whereof is 24.2; which added together, make 72.6 Yards, the Content at Work and half.

NOTE, That you muſt make Deductions for all Window-Lights; but you muſt meaſure the Window-Boards, Sopheta-Boards, and Cheeks, by themſelves.

§ V. *Of* PAINTERS Work.

THE taking the Dimenſions of Painters Work, is the ſame as that of Joyners, by girting over the Mouldings and Swelling Pannels, in taking the Height; and it is but Reaſon that they ſhould be paid for that on which their Time and Colour are both expended. The Dimenſions thus taken, the caſting up, and reducing Feet into Yards, is altogether the ſame as the Joyners Work; but the Painter never requires Work and half, but reckons his Work once, twice, or thrice colour'd over.

Only

Only take Notice, that Window-Lights, Window-Bars, Casements, and such like Things, they do at so much *per* Piece.

Example. If a Room be painted, whose Height (being girt over the Mouldings) is 16 Feet 6 Inches, and the Compass of the Room 97 Feet 9 Inches, how many Yards are in that Room?

Multiply 97 Feet 9 Inches by 16 Feet 6 Inches, and the Product is 1612 Feet 10 Inches 6 Parts; which being divided by 9 the Quotient is 179 Yards and 1 Foot.

```
[illegible] I.
  97   9
  16   6
 ---------
 584
  98
  48
 ---------
9)1612 10 6
 ---------
   179  1
```

```
   97.75
   16.5
 --------
   48875
  58650
  9775
 --------
 1612.875
```

Facit 179 Yards 1 Foot.

By Scale *and* Compasses.

Extend the Compasses from 9 to 16.5, that Extent will reach from 97.75 to 179.1 Yards.

§ VI. *Of* GLASIERS.

GLASIERS do measure their Work by the Foot Square; so that the Length and Breadth of a Pane of Glass in Feet, being multiply'd into each other, produceth the Content

NOTE, That Glaſiers do uſually take their Dimenſions of a quarter of an Inch; and in multiplying Feet, Inches, and Parts, the Inch is divided into 12 Parts, as the Foot is, and each Part ſubdivided into 12, *&c.*

Example 1. If a Pane of Glaſs be 4 Feet 8 Inches and 3 Quarters long, and 1 Foot 4 Inches 1 Quarter broad, how many Feet of Glaſs in that Pane?

The Decimal of $\left\{\begin{matrix} 8 \text{ Inches } \frac{3}{4} \\ 4 \text{ Inches } \frac{1}{4} \end{matrix}\right\}$ is $\left\{\begin{matrix} .729 \\ .354 \end{matrix}\right.$

F.	I.	P.				
4	8	9				4.729
1	4	3				1.354
4	8	9				18916
1	6	11	0			23645
	1	2	2	3		14187
						4729
6	4	10	2	3		6.403066

Anſwer, 6 Feet 4 Inches.

By Scale *and* Compaſſes.

Extend the Compaſſes from 1 to 1.354, that Extent will reach from 4.729 to 6.4 Feet, the Content.

Example 2. If there be 8 Panes of Glaſs, each 4 Feet 7 Inches 3 Quarters long, and 1 Foot 5 Inches 1 Quarter broad, how many Feet of Glaſs is contain'd in the ſaid 8 Panes?

The Decimal of $\left\{\begin{matrix} 7 \text{ Inches } \frac{3}{4} \\ 5 \text{ Inches } \frac{1}{4} \end{matrix}\right\}$ is $\left\{\begin{matrix} .646 \\ .437 \end{matrix}\right.$

F. I. P.	
4 7 9	4.646
1 5 3	1.437
	32522
4 7 9	13938
1 11 2 9	18584
1 1 11 3	4646
6 8 1 8 3	6.676902
8	8
53 5 1 6 0	53.410416

Facit 53 Feet 5 Inches.

By Scale *and* Compasses.

Extend the Compasses from 1 to 1.437, that Extent will reach from 4.646 to 6.676; then extend the Compasses from 1 to 8, that Extent will reach from 6.676 to 53.4, the Content.

Example 3. If there be 16 Panes of Glass, each 4 Feet 5 Inches and a half long, and 1 Foot 4 Inches 3 Quarters broad, how many Feet of Glass is contain'd therein?

F. I. P.	
4 5 6	4.458
1 4 9	1.395
	2229
4 5 6	40122
1 5 10 0	13374
3 4 1 6	4458
6 2 8 1 6	6.218910
4	4
24 10 8 6 0	24.87564
4	4
99 6 10 0 0	99.50056

Facit 99 Feet 6 Inches.

NOTE, That instead of multiplying by 16, I have multiply'd by 4 twice, because 4 times 4 is 16.

By Scale *and* Compasses.

Extend the Compasses from 1 to 1.395, that Extent will reach from 4.458 to 6.219; then extend the Compasses from 1 to 16, that Extent will reach from 6.219 to 99.5 Feet, the Content.

NOTE, That when Windows have half Rounds at the Top, they measure them at the full Height, as if they were square. Also round or oval Windows are measur'd at the full Length and Breadth of their Diameters. Likewise Crocket-Windows in Stone-work are all measur'd by their full Squares. And there is Reason for so doing; for the Trouble in taking their Dimensions to work by, the Waste of Glass in working, and the Time expended in setting up, is far more than the Glass can be valu'd at.

§ VII. *Of* MASONS Work.

MASONS do measure their Work sometimes by the Foot Solid, sometimes by the Foot superficial; and in some Places they measure their Walling by the Rod, that is 21 Feet long, and 3 Feet high, which is 63 square Feet.

Examples of each are as follow.

Example 1. If a Wall be 97 Feet 5 Inches long, 18 Feet 3 Inches high, and 2 Feet 3 Inches thick, how many solid Feet are contain'd in that Wall?

F.	I.			
97	5			97.417
18	3			18.25
				487085
776				194834
97				779336
24	4	3		97417
6	0	0		
1	6	0		1777.86025
				2.25
1777	10	3		
2	3			888930125
				355572050
3555	8	6		355572050
444	5	6	9	
				4000.1855625
4000	2	0	9	

Multiply the Length, Height, and Thickneſs together, and the laſt Product is 4000 Feet 2 Inches, the ſolid Feet contain'd in the Wall.

By Scale *and* Compaſſes.

Extend the Compaſſes from 1 to 18.25, that Extent will reach from 97.417 to 1777.86; then extend from 1 to 1777.86, that Extent will reach from 2.25 to 4000.18, the ſolid Content.

Example 2. If a Wall be 107 Feet 9 Inches long, and 20 Feet 6 Inches high, how many Feet ſuperficial is contain'd therein?

F.	I.		
107	9		107.75
20	6		20.5
2155	0		53875
53	10	6	21550
2208	10	6	2208.875

Facit 2208 Feet 10 Inches.

By Scale and Compasses.

Extend the Compasses from 1 to 107.75, that Extent will reach from 20.5 to 2208.875, the superficial Feet.

Example 3. If a Wall be 112 Feet 3 Inches long, and 16 Feet 6 Inches high, how many Rods are contain'd therein?

```
  F.   I.
 112   3
  16   6
 ---------
 676   0
112
  56   1  6
 ---------
1852   1  6
```

```
      112.25
        16.5
     -------
       56125
      67350
     11225
     -------
63)1852.125(29
     ---
     592
     ---
      25
```

Facit 29 Rods 25 Feet.

By Scale and Compasses.

Extend the Compasses from 63 to 16.5, that Extent will reach from 112.25 to 29.4 Rods, the Content.

CHAP.

CHAP. IV.

The Meafuring of BOARD *and* TIMBER.

§ I. *Of* BOARD-MEASURE.

TO meafure a Board, is no other but to meafure a long Square.

Example 1. If a Board be 16 Inches broad, and 13 Feet long, how many Feet is contain'd therein?

Multiply 16 by 13, and the Product is 208; which divided by 12, gives 17 Feet, and 4 remains, which is a third Part of a Foot.

Or thus: Multiply 156 (the Length in Inches) by 16, and the Product is 2496; which divided by 144, the Quotient is 17 feet, and 48 remains, which is a third Part of 144, the fame as before.

As

$$\begin{array}{r} \text{As } 12 : 13 :: 16 \\ 13 \\ \hline 48 \\ 16 \\ \hline 12)208 \\ \hline 17\tfrac{4}{12} \end{array}$$

$$\begin{array}{r} \text{Or, As } 144 : 156 :: 16 \\ 16 \\ \hline 936 \\ 156 \\ \hline 144)2496(17\tfrac{48}{144} \\ \hline 1056 \\ \hline 48 \end{array}$$

By Scale *and* Compaſſes.

Extend the Compaſſes from 12 to 13, that Extent will reach from 16 to 17 $\frac{1}{3}$ Feet, the Content.

Or, Extend from 144 to 156, (the Length in Inches) that Extent will reach from 16 to 17 $\frac{1}{3}$ Feet, the Content.

Example 2. If a Board be 19 Inches broad, how many Inches in Length will make a Foot?

Divide 144 by 12, and the Quotient is 7.58 very near; and ſo many Inches in Length, if a Board be 19 Inches broad, will make a Foot.

Inch. Inch. Inch. Inch.
As 19 : 144 : : 1 : 7.58 *fere.*

Extend the Compaſſes from 19 to 144, that Exte... will reach from 1 to 7.58; that is, 7 Inches and ſomething more than a half.

half. So, if a Board be 19 Inches broad, if you take 7 Inches and a little more than a half with your Compasses from a Scale of Inches, and run that Extent along the Board, from End to End, you may find how many Feet that Board contains, or you may cut off from that Board any Number of Feet desir'd.

For this purpose, there is a Line upon most ordinary Joint-Rules, with a little Table plac'd upon the End of all such Numbers as exceed the Length of the Rule, as in this little Table annex'd.

I.	0	0	0	0	5	0	8½	6	Long
F.	12	6	4	3	2	2	1	1	Long
I.	1	2	3	4	5	6	7	8	Broad

Here you see, if the Breadth be one Inch, the Length must be 12 Feet; if 2 Inches, the Length is 6 Feet; if 5 Inches broad, the Length is 2 Feet 5 Inches, *&c.*

The rest of the Lengths are express'd in the Line, thus: If the Breadth be 9 Inches, you will find it against 16 Inches, counted from the other End of the Rule; if the Breadth be 11 Inches, then a little above 13 Inches will be the Length of a Foot, *&c.*

§ II. *Of* SQUAR'D TIMBER.

BY squar'd Timber is here meant all such as have equal Bases, and the Sides strait and parallel. The Rules for measuring all such Solids, are shew'd in Section II. of Chap. 2; to which I refer you.

Example.

Example 1. If a Piece of Timber be 1 Foot 3 Inches (or 15 Inches) square, and 18 Feet long, how many solid Feet are contain'd therein?

```
       15
       15              F.  I.
      ----              1   3
       75               1   3
      15               -------
      ----              1   3
      225                   3   9
       18              ----------
      ----              1   6   9
     1800                       6
     225               ----------
     ----               9   4   6
144)4050(28.125                 3
    ----               ----------
    1170               28   1   6
     180
      360
       720
```

Answer, 28 Feet and half a Quarter.

Here, instead of multiplying by 18, (where I wrought by Feet and Inches) I multiply'd by 6, and then by 3, because 3 times 6 is 18.

Example 2. If a Piece of squar'd Timber be 2 Feet 9 Inches deep, and 1 Foot 7 Inches broad, and 16 Feet 9 Inches long, how many Feet of Timber are in that Piece?

Multiply the Depth, Breadth, and Length together, and the Product will be the Content.

33	F. I.
19	2 9
———	1 7
297	———
33	2 9
———	1 7 3
627	———
16.75	4 4 3
———	16 9
3135	———
4389	69 8 0
3762	3 3 2 3
627	———
———	72 11 2 3
144)10502.25(72.93	
———	
422	
1342	
465	
———	
33	

Answer, 72 Feet 11 Inches; or 72 Feet 93 Parts.

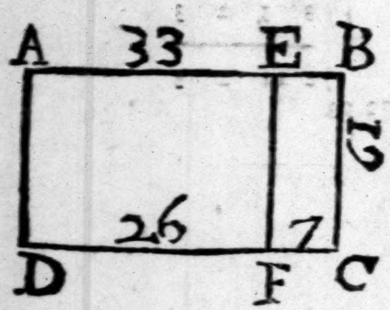
By Scale *and* Compasses.

For the first Example, extend the Compasses from 12 to 15 Inches, (the Side of the Square) that Extent will reach from 18 Feet, (the Length, being twice turn'd over) to 28 Feet and something more.

For the second Example, find a mean Proportional between 19 Inches and 33 Inches, by dividing the Space between them into two equal Parts; and the Compass Point will rest upon 25, which is a mean Proportional between 19 and 33.

Then extend the Compasses from 12 to 25, (the Proportional found) that Extent will reach (being twice turn'd over) from 16.75 Feet, the Length, to 72.93 Feet, the Content.

A common Error is committed, for want of Art, in meaſuring theſe laſt ſorts of Solids, by adding the Depth and Breadth together, and taking half for the Side of a mean Square. This Error, tho' it be but ſmall, when the Depth and Breadth are pretty near equal; yet if the Difference be great the Error is very conſiderable; for the Piece of Timber, thus meaſur'd, will be more than the Truth, by a Piece, whoſe Length is equal to the Length of the Piece of Timber to be meaſur'd, and the Square equal to half the Difference of the Breadth and Depth, as I ſhall here demonſtrate.



I ſay, the Square GHIK is greater than the Parallelogram ABCD, by the little Square OHPL; for the Parallelogram QPIK is equal to the Parallelogram AEFD; and the Parallelogram GOLQ is equal to the Parallelogram EBCF. Therefore the Squ. is greater than the Parallelogram by the little Square OHPL. Which was to be prov'd.

Otherwiſe, you may prove it by Numbers, thus; the Sum 33 and 19 is 52; the Half thereof is 26; the Square of 26 is 676; and the Product of the Depth and Breadth is 627; the Difference of theſe two is 49, equal to the Square of half the Difference; for the Difference between 33 and 19 is 14, the half thereof is 7, whoſe Square is 49. Which was to be prov'd.

Now, if this 49 be multiply'd by the Length of the Piece, and that Product divided by 144, to bring it to Feet, and thoſe Feet added to the true Content, the Sum will be equal to the Content found by the falſe Way mention'd.

See

See the WORK of both.

```
     33 Depth.         16.75 the Length.
     19 Breadth           49 the Sq. of half Diff
    ---                -----
     52 Sum.           15075
    ---                6700
     26 half,          -----
     26              4)820.75(5.69
    ---                -----
    156                1007
     52                1435
    ---                -----
    676                 139
  16.75
    ---
   3380
   4732
  4056
  676
  --------
144)11323.00(78.63
  -----
  1243
    910
     460
      28
```

Feet.
To 72.83 the true Content
Add 5.69 the Part superfluous.

Sum. 78.62 equal to the Content by the false Way.

By

By Feet *and* Inches.

F. I.
0 7
0 7

0 4 1
16 9

5 5 4
3 0 9

5 8 4 9 Part ſuperficious.
72 11 2 3 true Content add.

78 7 7 0 equal to the Content by the falſe Way.

F. I.
2 2
2 2

4 4
4 4

4 8 4
16 9

75 1 4
3 6 3

Falſe C. 78 7 7

To find how much in Length makes a Foot of any Squar'd Timber.

Always divide 1728 (the ſolid Inches in a Foot) by the Area of the Baſe; the Quotient is the Length of a Foot.

This Rule is general for all Timber, which is of equal Thickneſs from End to End, whether it be Square, Triangular, Multangular, or round.

Example 1. If a Piece of Timber be 18 Inches ſquare, how much in Length will make a Foot Solid?

18
18

144
18

324)1728($5\frac{1}{3}$
1620
108

Anſwer, 5 Inches and one third.

By

By Scale *and* Compasses.

Extend the Compasses from 1 to 18, that Extent will reach from 18 to 324, the Square or Area of the Base; then extend from 324 to 1728, that Extent will reach down from 1 to 5 Inches and $\frac{1}{3}$ of an Inch.

Or thus: Extend the Compasses from 18 to 41.569, that Extent, turn'd twice over from 1, will at last fall upon 5. $\frac{1}{3}$, as before.

Note, That 41.569 is the square Root of 1728.

Example 2. If a Piece of Timber be 22 Inches deep, and 15 Inches broad, how much in Length will make a Foot?

```
        22
        15
       ---
       110
        22
       ---
   330)1728(5.23
       ----
        780
       1200
       ----
        210
```

Answer, 5 Inches and . 23 Parts.

By Scale *and* Compasses.

Extend the Compasses from 1 to 15, that Extent will reach from 22 to 330; then extend from 330 to 1728, that Extent will reach from 1 to 5. 23 Inches, the Length of a Foot.

There

There is a Line for this Purpoſe upon moſt ordinary Rules, with a little Table at the End of all ſuch Numbers as exceed the Length of the Rule, ſuch as this annex'd.

0	0	0	0	9	0	11	3	9	Inches.
144	36	16	9	5	4	2	2	1	Feet.
1	2	3	4	5	6	7	8	9	Side of Sq.

Here you ſee, if the Side of the Square be 1, the Length muſt be 144 Feet; if two Inches be the Side of the Square, it muſt be 36 Feet in Length, to make a ſolid Foot, *&c.*

If the Side of the Square be not in the little Table, you will find it upon the Line; thus, if the Side of the Square be 16 Inches, you will find it againſt 6 Inches and 7 Tenths, counted from the other End of the Rule.

Then, if you take the Length of a Foot from the Line of Inches with your Compaſſes, and run the Compaſſes along the Piece, from End to End, you will find how many Feet are contain'd in that Piece; or you may cut off any Number of ſolid Feet that ſhall be deſir'd; but if the Sides of the Piece be unequal, find a mean proportional Number, as is before taught, by dividing the Diſtance upon the Line of Numbers into two equal Parts: Thus, if the Breadth be 25 Inches, and the Depth 9 Inches, divide the Space upon the Line of Numbers, into two equal Parts, and you will find the middle Point at 15; ſo is 15 Inches the geometrical mean Proportional ſought; then, if you look for 15 upon the Line abovemention'd, that 7 Inches, and a little above half, will be the Length of a Foot.

§ III. *Of unequal'd* Squar'd Timber.

BY unequal ſquar'd Timber, I mean all ſuch as have unequal Baſes; that is ſuch as is thicker at one End than at the other; and ſuch are moſt Timber-Trees, when they are hewn, and brought to their Squares.

The

The usual Way to measure such Timber, is to take a Square about the Middle of the Piece, which they take to be a mean Square: This Way, when the Piece is pretty near as thick at one End as at the other, is something near the Truth; but when there is a great Disproportion between the Ends of the Piece, the Error is considerable. All such Solids being the Frustrums of Pyramids, the true Way of measuring them must be by Sect. VII. Chap. 2. I shall give an Example or two, which I will work both by the true and false Ways, whereby you will see the Difference.

Example 1. If a Piece of Timber be 25 Inches square at the greater End, and 9 Inches square at the lesser End, and 20 Feet long, how many Feet of Timber is in that Tree?

```
     25
      9
     --
Sum  34
     --
half 17 the Side of the Square in the middle.
     17
     --
    119
    17
    ---
    289
     20
   ----
144)5710(40.13
   ----
    0200
      560
     ----
      128

Answ. 40.13.
by the false way.
```

By Rule II, Sect. VII, Chap. II

```
  25          25
   9           9
 ---          --
 225          16 Diff. of the Sides.
              16
              --
              96
              16
             ---
           3)256 the Square.
              85. 333
             225
             ---------
             310. 333
```

```
      310.3333
            20
    ------------
144)6206.6660(43.101
    ---------
     446
      146
       266
      ----
       122
```

Anſwer, 43.101 Feet, by the true Way; ſo that there is near 3 Feet difference.

By Scale *and* Compaſſes.

Extend from 1 to 9, that Extent will reach from 25 (the ſame Way) to 225, the Rectangle of the Sides of the two Baſes; then the Difference between the ſaid Sides, is 16: Extend from 3 to 16, that Extent will reach from 16 to 85.333, a third Part of the Square; which added to 225, the Sum is 310.333, a mean Area: Then extend from 144 to 310.333, that Extent will reach from 20 (the Length) to 43.1 Foot, the Content, the true Way.

Extend the Compaſſes from 12 to 17, (the Side of the middle Square) that Extent will reach from 20, (the Length, being twice turn'd over) to 40.1 Foot, the Content by the falſe Way.

Example 2. If a Piece of Timber be 32 Inches broad and 20 Inches deep at the greater End, and 10 Inches broad and 6 deep at the leſſer End, and 18 Foot long; how many Feet of Timber are in that Piece?

Rule I, Sect. VII, Chap. II.

```
 32        6
  20       10
 ---      ---
 640       60
   60
 ----
 38400
```

38400

```
. . .
38400  .........  ( 195.959 mean Proportional.
1                   640. the greater Base.
                    60. the lesser Base
-------             ---------
29) 284             895.959 the Sum.
 385)2300                 6 Height.
 3906)35000
 39185)231900       ---------
 391909)3597500     5375.754
              144)5375.754(37.33
                -----
                1055
                  477
                  455
                 -----
                   23
                 -----
Add. { 32          20 } Add.
     { 10           6 }
     ----          ---
Sum    42          26 Sum.
     ----          ---
Half   21          13 Half.
       13
      ----
       63
      21
      ----
      273 Area in the Middle.
       18 Length.
     -----
     2184
     273
     -----
  144)4914(34.12
     -----
      594
       180
        360
       -----
        72
```

Feet.

Answer, { Content the true Way 37.33
        { Content the false Way 34.12

By

By Scale *and* Compasses.

Extend the Compasses from 1 to 20, that Extent will reach from 32 to 640, the Area of the greater Base.

Then extend from 1 to 10, that Extent will reach from 6 to 60, the Area of the lesser Base: Then extend from 1 to 60, that Extent will reach from 640 to 38400, the Product of the two Areas: Find the square Root thereof, by dividing the Space between 1 and 38400 into two equal Parts, so you will find the middle Point at 195.959, the Root sought; which is a mean Proportional between the greater and the lesser Areas: Then add the mean Proportional and two Areas together, and the Sum is 895.959; which multiply'd by 6, (a third Part of the Length) by extending from 1 to 6, that Extent will reach from 895.959 to 5375.75; Then extend from 144 to 5375.75, and that Extent will reach from 1 to 37.33 Feet the true Content.

For this false Way, half the Sum of the Breadth is 21, which is the Breadth in the Middle; and half the Sum of the Depth is 13: Extend from 1 to 13, that Extent will reach from 21 to 273, the Area of the middle Base. Then extend from 144 to 273, that Extent will reach from 18 (the Length) to 34.12, the Content the false Way.

§ IV. *Of Round Timber, whose Bases are equal.*

THE usual Way to measure round Timber-Trees, is to girt them about the Middle with a String, and take the fourth Part of that Girt for the Side of a Square, by which they measure the Piece of Timber as if it was Square.

But that this is an Error, I shall make appear as follows. If the Circumference of a Circle be 1, the Area will be .07958; then the fourth Part of 1 is .25, which squar'd makes .0625, this they take for a mean Area, instead of .07958: Therefore the true Content always bears such Proportion to the Content found

found by the aforesaid customary false Way, as .07958 to .0625; which is nearly as 23 to 18; so that in measuring by that customary false Way, there is above the one fifth Part lost, of what the ttue Content ought to be.

This Error, tho' it has been so often confuted, yet is it grown so customary in all Places, that there is little Hopes of my prevailing with Men that are so wedded to it, to embrace the Truth; I shall therefore, in the following Examples, shew how to work both the true Way, and also the false or Customary Way.

Example 1. If a Piece of Timber be 96 Inches in Circumference or Girth. and 18 Feet long, how many Feet of Timber is contain'd therein?

A 4th Part of 96 is 24

```
    24
    24
   ----
    96
   48
   ----
   576 Area Base.
    18
   ----
  4608
  576
 ------
144)10368(72
   1008
   ----
    288
    288
   ----
    ...
```

Content the false Way 72 Feet.

Or thus,

```
 F.  I.
 2   0
 2   0
 ------
 4   0
18
 ------
72   0
```

Then the true Way.

```
       96
       96
      ----
      576
     864
     ------
     9216
     .07958
     -------
     73728
    46080
   82944
  64512
  ----------
  733.40928 the Area by Prob. 5, Sect. IX. Chap. 2.
         18
  ----------
  586727424
  73340928
  ----------
144)13201.36704(91.67
   ----------
    241
     973        The true Content 91.67 Feet.
    1096
     ----
      88
```

By Scale *and* Compaſſes.

Extend from 12 to 24. (the fourth Part of the Girth) that Extent turn'd twice over from 18 Feet, (the Length) will at laſt fall upon 72 Feet, the Content the cuſtomary Way.

Extend from 42.54 to 96, (the Girth) that Extent will reach from 18 Feet turn'd twice over, to 91.67 Feet, the true Content.

Example.

Example 2. If a piece of Timber be 86 Inches Girth, and 20 Feet long, how many Feet are contain'd therein?

The fourth Part of 86 is 21.5

F.	I.	P.	
1	9	6	
1	9	6	
1	9	6	
1	4	1	6
	0	10	9
3	2	6	3
			20
64	2	5	0

```
       21.5
       21.5
      ------
       1075
       215
      430
      ------
      462.25
          20
      ------
144)9245.00(64.2
      605
      290
       20
```

The Content the falſe Way 64.2 Feet.

By the true Way.

```
  86
  86
 ----
 516
688
 ----
7396
```

```
            7396
          .07958
          ------
           59168
          36980
         66564
        51772
        ---------
        588.57368
               20
        ---------
    144)11771.47360(81.74
        ----
         251
         1074
          667
         ----
           91
```

The true Content 81, 74 Feet.

By Scale *and* Compasses.

Extend from 12 to 21. 5, that Extent turn'd twice over from 20, will reach at last to 64.2 Feet, the Content the false Way.

Extend from 42.54 to 86, that Extent turn'd twice over from 20, will at last fall upon 81.74 Feet, the true Content.

These Cylindrical Proportions may be very easily wrought upon the Line of Numbers.

Problem 1. Having the Diameter of a Cylinder in Inches, to find the Length of a Foot.

Suppose the Diameter 22.6 Inches.

As 22.6 : to 46.9 : : so is 1 to a fourth Number: and that to the Length of a Foot in Inches, 4. 3.

Extend the Compasses from 22.6 to 46.9 that Extent will reach from 1 to a fourth Number; then turn them over again, and that will reach to 4.3 Inches.

Problem 8. Having the Diameter in Foot-Measure, to find the Length of a Foot in Foot-Measure.

Suppose

Suppose the Diameter 1.88 Feet.

Then, as 1.88 : to 1.128 : : so is 1 : to a fourth Number; and so is that to the Length of a Foot in Foot-Measure .358.

Extend the Compasses from 1.88 to 1.128, that Extent turn'd twice from 1, will reach to .358 Parts of a Foot.

Problem 3. Having the Circumference in Inches, to find the Length of a Foot in Inches.

Suppose the Circumference 71 Inches.

Then, as 71 : to 147.36 : : so 1 to a fourth Number; and so is that to the Length of a Foot in Inches 4.3.

Extend the Compasses from 71 to 147.36, that Extent turn'd twice from 1, will reach to 4.3 Inches. the Length of a Foot.

Problem 4. Having the Circumference in Foot-Measure, to find the Length of a Foot in Foot-Measure.

Suppose the Circumference 5.92 Feet.

Then, as 5.92 : to 3.545 : : so is 1 : to a fourth Number; and so is that to the Length of a Foot in Foot-Measure .358.

Extend the Compasses from 5.92 to 3.545, that Extent turn'd twice over from 1, will fall upon .358 Parts of a Foot.

Problem 5. Having the Diameter in Inches, and the Length in Inches, to find the Content in Inches.

Suppose the Diameter is 22.6 Inches, and the Length is 156 Inches, or 13 Feet.

Then, as 1.128 : to 22.6 : : so is 156 : to a fourth Number; and so is that to the Content in Inches, 62674.

Extend the Compasses from 1.128 to 22.6, that Extent, turn'd twice from 156, will fall upon 62674 Inches, the Content.

Note, That 1.128 is the Diameter when the Side of the Square equal is 1.

Problem 6. Having the Diameter in Foot Measure, and Length in Feet, to find the Content in Feet.

Suppose the Diameter 1.88 Feet, and the Length 13 Feet.

Then, as 1.128 : to 1.88 : : so is 13 to a fourth Number; and so is that to the Content in Feet, 36.27.

Extend

Extend from 1.128 to 1.88, that Extent turn'd twice from 13, will fall upon 36.27

Problem 7. Having the Diameter in Inches, and Length in Inches, to find the Content in Feet.

Suppoſe the Diameter 22.6 Inches, and the Length 156 Inches,

Then, as 46.9 : is to 22.6 : ſo is 156 : to a fourth Number, and ſo is that to the Content in Feet, 36.27

Extend from 46.9 to 22.6, that Extent turn'd twice from 156, will fall upon 36.27 Feet, the Content.

Note, That 46.9 is the Diameter of a Circle, whoſe Area is 1728.

Problem. 8. Having the Diameter in Inches, and Length in Feet, to find the Content in Feet.

Suppoſe the Diameter 22.6 Inches, and the Length 13 Feet.

Then, as 13.54 : to 22.6 : : ſo is 13 to a fourth Number; and ſo is that to the Content in Feet 36.27.

Extend from 13.54 to 22.6, that Extent, turn'd twice from 13, will fall upon 36.27.

Note, That 13.54 is the Diameter of a Circle, when the Area is 144.

Problem 9. Having the Circumference in Inches, and Length in Inches, to find the Content in Inches.

Suppoſe the Circumference 71, and the Length 156 Inches;

Then as 3.545 : is to 71 : : ſo is 156 to a fourth Number; and ſo is that to 62674, the Content in Inches.

Extend the Compaſſes from 3.545 to 71, that Extent, turn'd twice from 156, will fall upon 62674, the Content.

Note, That 3.545 is Circumference, when the Side of the Square equal is 1.

Problem 10. Having the Circumference in Feet, and Length in Feet, to find the Content in Feet.

Suppoſe the Circumference 5.92 Feet, and Length 13 Feet;

Then, as 3.545 : to 5.92 : : ſo is 13 to a fourth Number; and ſo is that to 36.27.

Extend from 3.545 to 5.92, that Extent, turn'd twice from 13, will fall upon 36.27 Feet, the Content,

Problem 11. Having the Circumference in Inches, and Length in Inches, to find the Content in Feet.

Suppose the Circumference 71 Inches, and Length 156 Inches; Then, as 147.36 : to 71 : : so is 156 to a fourth Number; and so is that to the Content in Feet 36.27.

Extend the Compasses from 147.36 to 71, that Extent turn'd twice from 156, will fall upon 36.27 Feet the Content.

Note, That 147.36 is the Circumference of a Circle, whose Area is 1728.

Problem 12. Having the Circumference in Inches, and Length in Feet, to find the Content in Feet.

Suppose the Circumference 71 Inches, and Length 13 Feet.

Then, as 42.54 : is to 71 : : so is 13 to a fourth Number: and so is that to the Content in Feet, 36.27.

Extend the Compasses from 42.54 to 71, that Extent turn'd twice from 13, will reach to 36.27 Feet, the Content.

Note, That 42.54 is the Circumference of a Circle, whose Area is 144.

§ V. *Of Round Timber, whose Bases are unequal.*

THE usual Way to measure round Timber, (as I said before) is to take a fourth Part of the Girth in the Middle of the Piece, for the Side of a mean Square. But this Way I have prov'd to be erroneous in Timber that is all the Way of an equal Thickness; and it must be much more so in Timber that is tapering, and the more tapering it is, the greater is the Error: For to the Error in the last Section, there is added the Error in the third Section; therefore, to measure all such Timber according to Art and Truth, such a Piece ought to be consider'd as a Frustrum of a Cone, and should be measur'd by the Rules given in Section VIII, Chapter II, by which Rules the following Examples are wrought.

Example

Example 1. If a piece of Timber be 9 Inches Diameter at the leffer End, and 36 Inches at the other End, and 24 Feet long, how many Feet of Timber is therein?

```
      36                 36 } Subtract.
       9                  9 }
     ---               ----
Rect. 324                27   Difference,
                         27
                        ---
                        189
                        54
                        ---
                      3)729  the Square.
                        ---
                        243  one third.
                        324  Rectangle add,
                        ---
                        567
                       .7854
                      -----
                       54978
                      47124
                     39270
                     -------
A Mean Area          445,3218
                          24
                    --------
                    17812872
                    8906436
                   ---------
                144)10687.7232(74.22
                   ------
                    607
                     317
                      292
                    ---
                      4

           Answer 74 .22 Feet.
```

Or

Or thus by Feet and Inches.

F.	I.		F.	I.		
3	0		2	3		Difference.
0	9		2	3		
2	3	Rect.	4	6		
			0	6	9	
			5	0	9	the Square.
			1	8	3	one Third.
			2	3	0	Rectangle added.
			3	11	3	a mean Square.

F. I. P.

Then, as 14 : to 11 : : so is 3 11 3 to the Area

```
          3  11  3
                11
       ------------
       7)43   3  9
       ------------
       2) 6   2  3
       ------------
          3   1  1  6
                    6
       ---------------
         18   6  9  0
                    4
       ---------------
         74   3  0  0
```

Here instead of dividing by 14, I divide by 7 and by 2, because twice 7 is 14.

And instead of multiplying by 24 Feet, the Length, I multiply by 6 and by 4, because 6 times 4 is 24.

By Scale and Compasses this is too troublesom.

Example 2. If a Piece of Timber be 136 Inches Circumference at one End, and 32 Inches Circumference at the other End, and 21 Feet long, how many Feet of Timber is contain'd in that Piece?

136

```
 136            136
  32             32
----           ----
 272            104 Difference.
408             104
----           ----
4352            416
               104
              -----
           3)10816 the Square.
            -------
             3605.333 one Third.
             4352     Rectangle add.
            ----------
             7957.333 a mean Circumf. ſquar'd
              .07958
            ----------
              63658664
             3978665
            71615997
           55701331
          --------------
           633.24456014 the mean Area.
                     21
          --------------
          63332445 6014
         126648912028
         ---------------
        13298.13576294

           144)13298.13(92.34
               ------
                338
                 501
                 693
                -----
                 117
```

Anſwer, 92.34 Feet.

By

By Feet and Inches thus;

```
F.  I.        F.  I.
11  4         8   8    Difference.
 2  8         8   8
--------      ---------
22  8         69  4
 7  6  8       5  9  4
--------      ---------
30  2  8   3) 75  1  4 the Square.
             ------------
              25  0  5  4
              30  2  8  0
             ------------
              55  3  1  4 the Sq. of the Circumf.

              F.  I.  P.  S.
As 88 : to 7 :: 55  3  1  4 : to the mean Area.
                          7
               -------------
          11) 386  9  9  4
               -------------
           8)  35  1 11  9
               -------------
                4  4  8 11 the mean Area.
                          7
               -------------
               30  9  2  5
                          3
               -------------
         Facit 92  3  7  3
```

§ VI.

§ VI. *Of the five regular Bodies.*

THESE Bodies may all be measur'd by the 4th Section of Chap. II, except it be the Cube, or Hexaedron, which is already measur'd in Section I. of that Chapter.

I. *Of the* TETRAEDRON.

A Tetraedron is a Solid contain'd under four equal and equilateral Triangles.

Let ABCD be a Tetraedron, whose Side is 12 Inches, the perpendicular Height 9.798 Inches.

By Sect. V, Chap. I, the Area of the Triangle will be found 62.352; a third Part of it is 20.7843 which multiply'd by the perpendicular Height, the Product is 203,641632 solid Inches, the Content.

10.392 the Perpendicular of the Triangle.
6 half the Side.

62.352 Area of the Triangle.

20.784 one third Part.
9.798 the perpendicular Height.

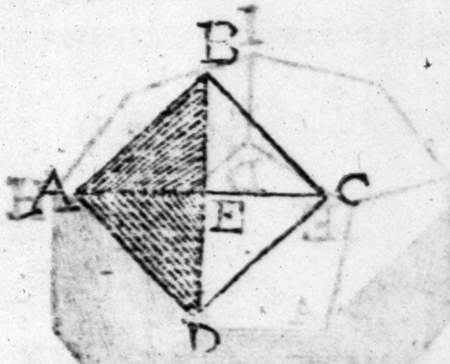
166272
187056
145488
187056

203.641632 the Solidity.

Th

The ſuperficial Content is four times the Area of the Triangle, *viz.* 249.408, Inches. becauſe there are 4 Triangles.

2. *Of the* OCTAEDRON.



The Octaedron is a Body contain'd under eight equal and equilateral Triangles.

Let ABCDE be an Octaedron, whoſe Side is 12 Inches; the Content ſolid and ſuperficial is requir'd.

An Octaedron is compos'd of two quadrangular Pyramids join'd together by their Baſes; therefore, if the Area of the the Baſe be multiply'd into a third Part of the Length of both Pyramids, the Product will be the ſolid Content.

5.6568 a third Part of the Length.
144 Area of the ſquare Baſe.

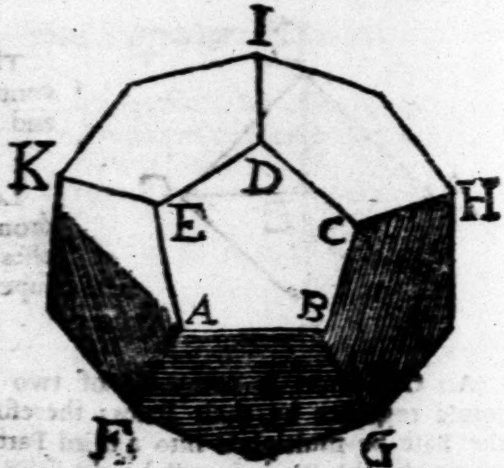
226272
226272
56568

814.5792 the Solidity.

The ſuperficial Content will be juſt double to that of the Tetraedron, *viz.* 498.816, becauſe the Side of this is ſuppos'd to be equal to the Side of that, and becauſe the Octaedron is contain'd under eight Triangles, and the Tetraedron but under four,

3. *Of the* DODECAEDRON,

The Dodecaedron is a ſolid Body, contain'd under twelve pentangular Planes.



Let ABCDEFG HIK be a Dodecaedron, each Side thereof being 12 Inches; the Content ſolid and ſuperficial is required.

The Solidity of the Dodecaedron is compos'd of 12 pentangled Pyramids, whoſe Vertices all meet in the Center. Therefore, if we find the Solidity of one of thoſe Pyramids, and multiply that by 12, that Product will be the Solidity of the Dodecaedron.

The Altitude of one of the pentangled Pyramids will be found to be 13.36219.

The Perpendicular of the Pentagon will be

8.258292
30 half Sum of the Sides.

247.748760 Area of the Pentagon.
60454.4 a third Part of 13.36219 inverted.

99099504
9909950
1238744
99099
1486

1103.48783 Content of one Pyramid.
12

13241.85396 the Solidity of the Dodecaedron.

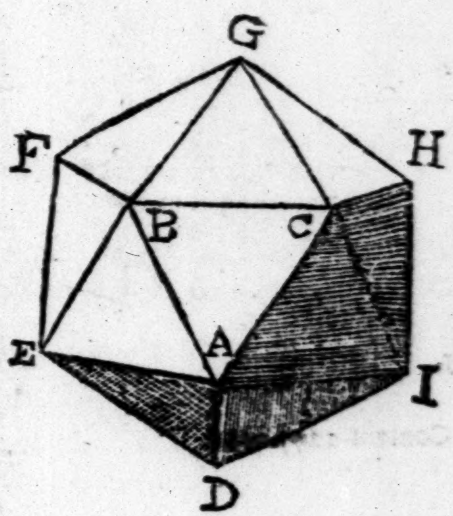
If

If the Area of the Pentagon be multiply'd by 12, the Product will be the superficial Content.

247.74876
12

2972.98512 the superficial Content.

4. *Of the* Icosaedron.



The Icosaedron is a solid Body, contain'd under twenty equal and equilateral Triangles.

Let ABCDEFGHI be an Icosaedron, each Side thereof being 12 Inches; the Content Solid and superficial is requir'd.

The Icosaedron is compos'd of twenty triangular Pyramids, with their Vertices all joyning in the Center.

Therefore, if the solid Content of one Pyramid be multiply'd by 20, the Product is the whole solid Content of the Icosaedron.

10.39224 the Perpendicular of the Triangle.
6 half the Side.

62.35344
20

1247.06880

3.0230456 the third Part of the Altit. of the Pyramid.
44353.26

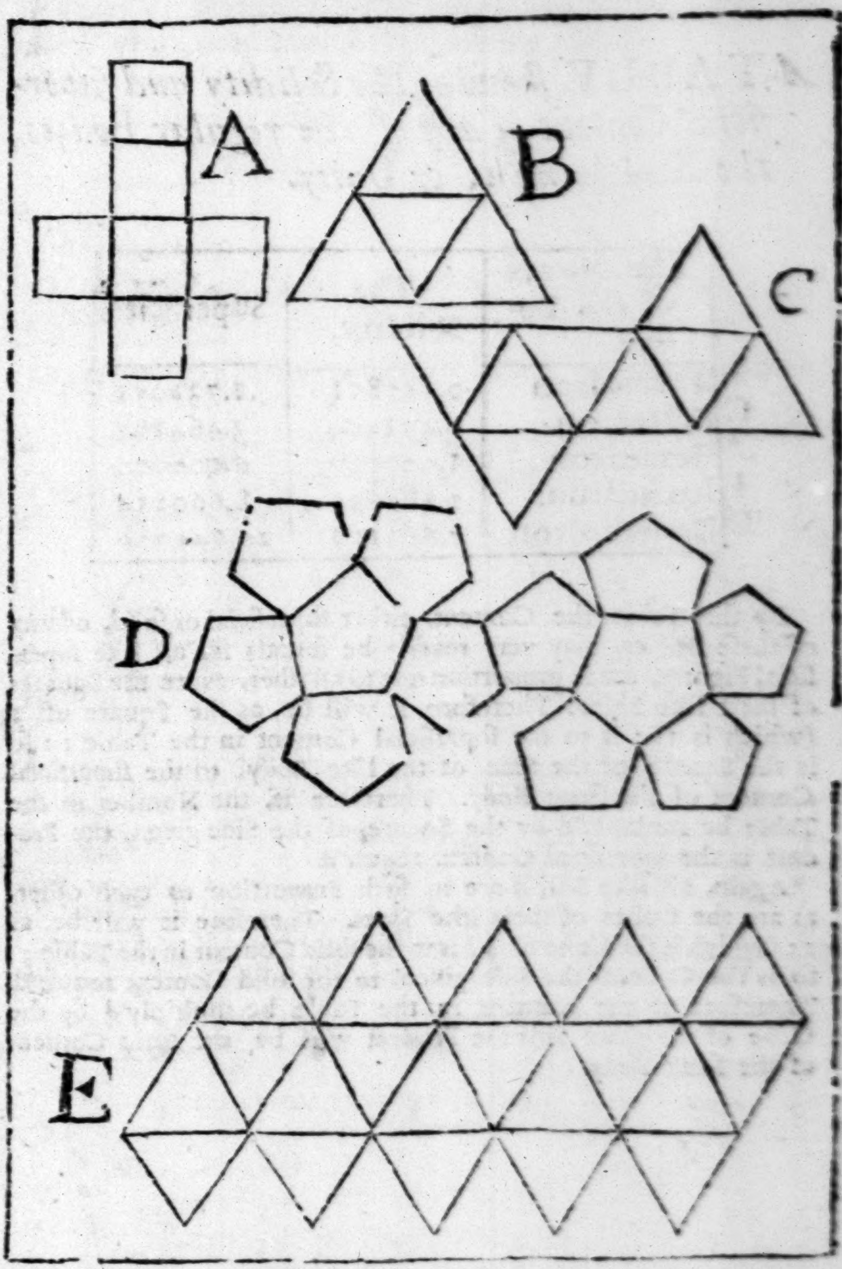
181382736
6046091
906914
151152
9069
1209
121

188.497292
20

3769.945840 the Solidity.

The superficial Content 1247.0688.

The



A. the Cube. B. Tetraedron. C. Octaedron. D. Dodecaedron. E. Icosaedron.

By these Figures you may cut these Bodies in fine Pastboard, cutting all the Lines half through, and so turn them up and glew them.

A TABLE *ſhewing the Solidity and ſuperficial Content of any of the regular Bodies, the Side being* 1, *or Unity.*

	The Names of the Bodies.	The Solidity.	Superficies.
B	Tetraedron	0.117851	1.732051
C	Octaedron.	0.471404	3.464102
A	Hexaedron	1.000000	6.000000
F	Icoſaedron.	2.181695	8.660254
D	Dodecaedron	7.663119	20.645729

By this Table, the Content, either ſuperficial or ſolid, of any of theſe Bodies, may very readily be found; for all like ſuperficial Figures, are in proportion one to another, as are the Squares of their like Sides: Therefore it will be, as the Square of 1 (which is 1) : is to the ſuperficial Content in the Table : : ſo is the Square of the Side of the like Body, to the ſuperficial Content of the ſame Body. Therefore if, the Number in the Table be multiply'd by the Square of the Side given, the Product is the ſuperficial Content requir'd.

Again, all like Solids are in ſuch Proportion to each other, as are the Cubes of their like Sides. Therefore it will be, as 1 : (which is the Cube of 1.) is to the ſolid Content in the Table : : ſo is the Cube of the Side given, to the ſolid Content requir'd. Therefore, if the Number in the Table be multiply'd by the Cube of the given Side the Product will be the ſolid Content of the ſame Body.

Example 1. If the Side of a Dodecaedron be 12 Inches (as before) what is the Content solid and superficial?

7.663119 the tabular Number.
1728 the Cube of the Side.

61304952
15326238
53641833
7663119

13241.869632 the solid Content nearly the same as before

20.645729 the tabular Number.
144 the Square of the Side.

82582916
82582916
20645729

2972.984976 the superficial Content.

By Scale *and* Compasses.

Extend from 1 to 12, (the Side) that Extent being turn'd three times over from 7.663119, will at last fall upon 13241.86, *&c.* the solid Content.

And if you apply the same Extent twice from 20.645729, it will at last fall upon 2972.98, *&c.* the superficial Content.

Example 2. If the Side of an Octaedron be 20 Inches, what is the Content solid and superficial?

.4714045 the tabular Number.
8000 the Cube of the Side.

3771.2369000 the solid Content.

3.464102 the tabular Number.
400 the Square of the Side.

1385.640800 the superficial Content.

By Scale *and* Compasses.

Extend from 1 to 20, that Extent turn'd three times over from .4714045, will at last fall upon 3771, 236, the solid Content. The same Extent, turn'd twice over from 3.464, *&c.* will at last fall upon 1385.64, the superficial Content.

§ VII. *How to measure any irregular Solid.*

IF you have any Piece of Wood or Stone that is craggy and uneven, and you desire to find the Solidity, put the Solid into any regular Vessel, as a Tub, a Cistern, or the like, and pour in as much Water as will just cover it; then take out the Solid, and measure how much the Fall of the Water is, and so find the Solidity of that Part of the Vessel.

Example. Suppose a Piece of Wood or Stone to be measur'd, and suppose a Tub 32 Inches Diameter, into which let the Stone or Wood be put, and cover'd with Water; then when the Solid is taken out, suppose the fall of the Water 14 Inches: square 32, and multiply the Square by .7854, the Product will be 804.2496, the Area of the Base; which multiply'd by 14, the Depth or Fall of the Water, and the Product is 11259.49, *&c.* which divided by 1728, the Quotient is 6.51 Feet; and so much is the solid Content requir'd.

CHAP.

CHAP. V.

Practical Questions in MEASURING.

Question 1. IF a Pavement be 47 Feet 9 Inches long, and 18 Feet 6 Inches broad, I demand how many Yards are contain'd therein?

F.	I			47.75
47	9			18.5
18	6			
376	0			23875
47				38200
23	10	6		4775
9	0	0		
4	6	0		9)883.375
				98.1
9)9883	4	6		

Answer, 98 Yards 1 Foot.

Quest. 2. There is a Room, whose Length is 215 Feet, and the Breadth 17.5 Feet, is to be pav'd with Stones, each 18 Inches square; I demand how many such Stones will pave it?

```
      21.5            1.5
      17.5            1.5
      ----            ---
      1075            75
     1505            15
     215             ---
     -----           2.25 Area of one Stone.
2.25)376.25(167
     -------
     1512
     1625
     ----
       50      Answer 167 Stones.
```

Quest. 3. There is a Room 109 Feet 9 Inches about, and 9 Feet 3 Inches high, which is all (except two Windows, each 6 Feet 6 Inches high, and 5 Feet 9 Inches broad) to be hung with Tapestry that is Ell broad; I desire to know how many Yards will hang the said Room.

From the Content of the Room, subtract the Content of the Windows, and divide the Remainder by the Square Feet in a Yard of Tapestry.

```
 3.75      109.75 Length                              5.75
 3           9.25 Breadth.                            6.5
 -----     -------                                   -----
 11.25      54875                                     2875
           21950                                     3450
          98775                                      -----
          ---------                                  37375
          1015.1875 Content of the Room.                 2
            74.75   Content of the Windows sub.      -----
          ---------                                  74.750
11.25)940.4375(83.59
      ---------
      4043
       6687
       10625
       -----
        500
     Answer 83.59 Yards.
```

Quest.

Quest. 4. If the Axis of a Globe be 27.5 Inches, I demand the Content solid and superficial?

```
          3.1416
            27.5
        --------
         157080
        219912
        62832
        --------
        86.39400 the Circumference.
           27.5  the Diameter.
        --------
        431970
       604758
      172788
      --------
     6)2375.8350 the superficial Content.
      ---------
       395.9725 a sixth Part.
          27.5
      ---------
      19798625
     27718075
     7919450
    ----------
 10889.24375 The Solidity in Inches.
```

Answer { 6.3 Feet solid. 16.49 Feet superficial.

Quest. 5. There is the Frustrum of a Globe, the Diameter of whose Base is 24 Inches, and the Altitude thereof is 10 Inches; what is the Content solid and superficial?

Find the Superficies as is directed in Pag. 169, and find the Solidity by the first Theorem in Pag. 180.

```
 24          .7854        .7854        20
 24            576          400        20
----        -------     ---------     ----
 96          47124      314,16000     400
48          54978
----        39270
576         --------
            452.3904 } add.
            314.16
            --------
```

766.550

766.5504 the Curve Superficies.
452.3904 the Base add.

1218.9408 } The whole superficial Content in Inches.

12 × 12 = 144
3

432
100 the Square of the Alt. add.

532 the Sum.
10 multiply by the Alt.

5320
.5236 multiply

31920
15960
10640
26600

2785.5520 the Solidity in Inches.

Quest. 6. If a Tree girt 18 Feet 6 Inches, and be 24 Feet long; how many Tuns of Timber are contain'd in that Tree?

F.	I.			
4)18	6			the Girth.
4	7	6		a 4th Part.
4	7	6		
18	6	0		
2	8	4	6	
	2	3	9	
21	4	8	3	
			6	
828	4	1	6	

Here I multiply by 6 and by 4, because 6 times 4 is 24.

```
       128  4  1  6
                  4
   ----------------
 4|0)51|3  4  6  0
   ---------
     12  33
```

Answer, 12 Tuns 33 Feet 4 Inches 6 Parts.

Note, that 40 Feet of Timber is a Tun, and 50 Feet a Load.
Note also, That 4 Feet broad, 4 Feet deep, and 8 Feet long, is a Cord of Fire-wood, that is 128 Cubical Feet.

Quest. 7. There is a Cellar to be dug by the Floor, whose Length is 33 Feet 7 Inches, and the Breadth is 18 Feet 9 Inches, and its Depth to be 5 Feet 9 Inches, I demand how many Floors of Earth are in that Cellar?

```
         F.  I.
         33  7 the Length.
         18  9 the Breadth.
       ---------
        264
        33
         16  9  6
          8  4  9
          9  0  0
          1  6  0
       ---------
        629  8  3
          5  9 the Depth.
       ---------
       3148  5  3
        814 10  1
        157  5  0
       ---------
 324)3620  8  4 (11
       ----
        380
       ----
         56
```

Answer, 11 Floors 56 Feet.

Note, That 18 Feet square, and a Foot deep, is a Floor of Earth, that is 324 solid Feet.

Quest.

Quest. 8. There is a Roof cover'd with Tiles, whose Depth on both Sides (with the usual Allowance at the Eaves) is 35 Feet 6 Inches, and the Length 48 Feet 9 Inches: how many Squares of Tiling are contain'd therein?

```
     F.  I.
     48  9
     35  6
    -------
    240
   144
     24  4  6
     17  6  0
      8  9  0
    ---------
  17|30  7  6
```

Answer, 17 Squares 30 Feet.

Quest. 9. There is a Cone, whose Diameter at the Base is 42 Inches, and the Perpendicular Height 94 Inches, and it is requir'd to cut off two solid Feet from the top End thereof; I demand what Length upon the Perpendicular must be cut off?

```
          42      1728        94
          42         2        94
          --      ----       ----
          84      3456       376
        168                 846
        ----                ----
        1764                8836 Square.
        .7854                 94
        -----               -----
        7056               35344
       8820               79524
     14112                ------
    12348                 830584 the Cube.
    ----------
    1385.4456
           94
    ----------
     55417824
   124690104
   -----------
 3)130231.8864
   -----------
    43410.6288
```

All

All the Solid Bodies are in triplicate Reason of their homologous Sides by *Euc.* 12, 12; 12. 18; and 11, 33; therefore it will be,

Solidity of the Cone. Cube Alt. Solidity of 2 Feet.
As 43410.6288 : 830584 :: 3456 :
3456

4983504
4152920
3322336
2491752

43410.6288)2870498304(66124 the Cube of the Length.

265860576
5396803
1055740
187528

66124(40.3
64

2124000 Resolvend.

12
48

492 Divisor.

120
4800

48120 Divisor.
64
1920
19200

1939264 Subtrahend.

184736000 Resolvend.

184736000

1212
489648

4897692 Divisor.

27
10908
1468944

147003507 Subtrahend.

377732493

Answer. The Length upon the Perpendicular must be 40.43 Inches. If it had been 3 Feet, the Length had been 46.29 Inches.

If two Feet was to be cut off from the Bottom, or greatest End, then from 43410.6288 subtract 3456, and the Remainder is 39954.6288. Then say,

As 43410.6288 : 830584 :: 39954.6288
830584

1598185152
3196370304
1997731440
11986388864
3196370304

43410.6288) 33185675407.2192(764459(91.4
.....
729

279823524
19359751 35459
1995500

259075 27
42022 243

2952 2457

2457

271
243

24571

10888000

273
24843

248703

Answer, It must be cut at 91.4 Inches from the Top, or 2.6 Inches from the Bottom.

Quest. 10. If a square Piece of Timber be 12 Feet long, and if the Side of the Square of the greater Base be 21 Inches, and the Side of the Square of the lesser Base be 3 Inches; how far must I measure from the greater End, to cut off 5 solid Feet?

First, Find the Length of the whole Pyramid, thus; the Difference between 21 and 3 is 18; then.

Diff. Length. great. Length.
As 18 : 12 :: 21 : 14,

So I find the whole Length of the Pyramid 14 Feet, or 168 Inches,

The solid Content of the whole Pyramid is 24696 Inches, and the solid Content of 5 Feet is 8640; which subtracted from 24696, there remains 16056 Inches. Then, the Cube of 168 (the Length) is 4741632. Then,

As 24696 : 4741632 : : 16056:

To 3082752, whose Cube Root is 145.54; subtract this Root from 168 (the Length) and there remains 22.46 Inches which is the Length of 5 solid Feet at the great End.

Quest. 11. Three Men bought a Grinding Stone of 40 Inches Diameter, which cost 20 Shillings; of which Sum, the first Man paid 9 Shillings, the second 6 Shillings, and the third 5 Shillings; I demand how much of the Stone each Man must grind down, proportionable to the Money he paid?

All Circles are in duplicate Reason of their Diameters, by *Eucl.* 12, 2.

Square the Semidiameter, which makes 400. Then,

s. s.
As 20 : 400 : : 9 : 180.

This 180 is the Square of the Semidiameter of the Circle belonging to the first Man.

s. s.
And, as 20 : 400 : : 6 : 120.

This 120 is the Square of the Semidiameter of the Circle belonging to the second.

s. s.
And, as 20 : 400 : : 5 : 100.

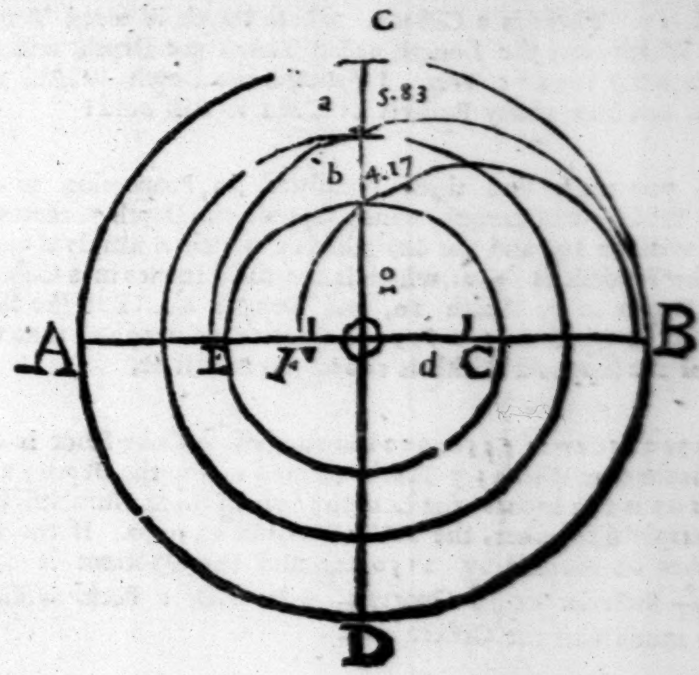
This 100 is the Square of the Semidiameter of the Circle belonging to the third.

Then from 400 (the Square of the Semidiameter of the Stone) subtract 180, and there remains 220, whose square Root is 14.83 Inches; which subtracted from 20 Inches, (the Semidiameter) there remains 5.17 Inches, which is the Breadth of the Ring, or Part of the Stone which must be ground down by the first.

Then, from 220 subtract 120, and there remains 100, whose square Root is 10; subtract that from 14.83, and there remains 4.83 Inches, the Breadth of the Ring, or Part to be ground down by the second Man. The third must grind down the Remainder, which is 10 Inches, the Square Root of 100.

This Question may very easily and speedily be performed Geometrically, as in the annex'd Scheme.

First,



First, upon the Centre ⊙ ſtrike the Circle ACBD, and croſs it at Right-Angles with the two Diameters AB and CD: Then divide the Semidiameter A⊙ (which ſuppoſe 20) in Proportion, as 9s. 6s. and 5s. (the ſeveral Sums paid by the three Men) by the Points E and F; ſo ſhall AE be 9, EF 6, and F⊙ 5: Then divide EB into 2 equal Parts in d, and upon d, as a Center, ſtrike the Semicircle EaB; and divide FB into 2 equal Parts in c, and upon c, as a Center, with the Radius cF, ſtrike the Semicircle FbB: So have you the Semidiameter ⊙C divided into three ſuch Parts as the Stone ought to be divided; and Circles ſtruck thro' thoſe Points, will ſhew how much each Man muſt grind for his Share.

Queſt. 12. A Gard'ner he had an upright Cone,
Out of which ſhould be cut him a Rolling-Stone,
The biggeſt that e'er it could make:
The Maſon he ſaid, that there was a Rule
For ſuch Sort of Work, but he had a thick Skull;
Now help him for Pity's Sake.

Anſwer, It muſt be cut at one third Part of the Altitude.

Quest. 13. There is a Cistern, whose Depth is seven Tenths of the Width, and the Length is six Times the Depth, and the solid Capacity is 36.75. Feet. I demand the Depth, Width, and Length, and how many Bushels of Corn it will hold?

First, you must find three Numbers, in Proportion to the Depth, Width, and Length, thus; suppose the Depth 7, then the Width will be 10, and the Length 42; which multiply'd together, the Product is 2940, which is the solid Inches in a Cistern, whose Depth is 7, Width 10, and Length 42. But the solid Inches in the Question are 635040 ($= 36.75 \times 1728$) then the Cube of the suppos'd Width is 1000. So it will be,

As 2940 : 1000 :: 635040 : 216000, whose Cube Root is 60, which is the true Width; 7 Tenths thereof is 42, the Depth; and 6 times 42 is 252 Inches, the Length; which three Numbers being multiply'd together, the Product will be 635040. If the solid Inches be divided by 2150.42, and the Quotient is 295 $\frac{66610}{215042}$ Bushels, or 36 Quarters, 7 Bushels, 1 Peck, 4 Pints And so much will the Cistern hold.

Quest. 14. *Suppose, Sir, a Bushel be exactly round*

Whose Depth being measur'd, 8 *Inches is found,*

If the Breadth 18 *Inches and Half you discover,*

This Bushel is legal all England *over.*

But a Workman would make one of another Frame;

Sev'n Inch and a Half must be the Depth of the same:

Now, Sir, of what Length must the Diameter be,

That it may with the former in Measure agree?

```
        18.5
        18.5
        ----
         925
        1480
        185
      ------
      342.25 the Square.
       .7854
     -------
      136900
     171125
    273800
   239575
  -----------
  268.803150
           8
  -----------
 2150.425200 the ſolid Inches in a Buſhel.

          7.5)2150.4252(286.72336
                  . . .
              -------
               650
                504
                 342
                  175
                   252
                    270
                     450
                    ----
                       0
```

```
.7854)286.72336 . . . (365.0666(19.107
                        1
 ——————              ————
   51103
   39793           29)265
     52360            261
       5236        ————
                   381)406
                       381
                      ————
                 38207)256600
```

Answer, The Diameter must be 19.107 Inches, if the Depth be 7.5 inches.

Quest. 15. *In the midst of a Meadow well stored with Grass.*
I took just an Acre to tether my Ass;
How long must the Cord be, that feeding all round,
He mayn't graze less nor more than his Acre of Ground?

By Problem 10, Section IX, Chap. I. find the Diameter of a Circle containing an Acre; half that will be the Length of the Cord.

The WORK.

```
660 Feet, the Length of an Acre.
 66 Feet, the Breadth of an Acre.
————
 3960
3960
——————
43560 the square Feet in an Acre.

        As 1 : 1.2732 :: 43560
                 43560
              ——————————
                 763920
                63660
               38196
              50928
              ————————
```

55460.

```
        . . .
      55460.5920(235.5 Diamet.
      4              ------
     ---             117.75 half.
   43)154
      129
     ----
  465)2560
     2325
    -----
 4705)23559
      23525
      -----
         34
```

Anſwer, The Cord muſt be 117 Feet and 9 Inches.

But in an *Iriſh* Acre is 70560 Feet *i.e.* 21 x 21 x 160 == 70560 then ſay as above.

```
As 1 : 1.2732 : : 70560
          70560
        -------
        7633920
        63660
      891240
      ---------
       . . .
      89836.9920(299.72 Diamet.
      4            -------
     ---           149.86 half.
   49)498
      441
    -----
  589)5736
      5301
    ------
 5987)43599
      41909
     -------
 5994)169020
```

Queſt. 16. A Maltſter has a Kiln that is 16 Feet 6 Inches ſquare; but he is minded to pull it down, and build a new one, that may be big enough to dry three times as much at a Time as the old one will do; I demand how much ſquare the New One muſt be?

```
       16.5
       16.5
      -----
        825
       990
      165
     ------
     272.25 the Area of the old one.
          3
     ------
     816.75(28.57
     4
     ----
   48)416
      384
     ----
  565)3275
      2825
     -----
 5707)45000
      39949
      -----
       5051
      -----
```

Anſwer. The Side of the new one muſt be 28 Feet, and near 7 Inches.

Queſt.

Quest. 17. If a round Cistern be 26.3 Inches Diameter, and 52.5 Inches deep, how many Inches Diameter must a Cistern be, to hold twice the Quantity, the Depth being the same? And how many Ale-Gallons will each Cistern hold?

```
       26.3
       26.3
      ------
        789
      1578
      526
    ---------
     691,69 the Square.
          2
    ---------
      . .
    1383.38(37.19
    9
    -----
   67)483
      469
    -----
  741)1438
       741
    ------
 7429)69700
      66861
     ------
       2839
     ------
```

The Diameter of the greater is 37.19 Inches.

691.69

```
        691.69 the Square of the lesser Cistern's Diameter.
         .7854
        ------
        276678
       345845
      553352
     484183
    ----------
     543.253326 Area of the Base.
           52.5
    -----------
      2716266630
     1086506652
    2716266630
    ---------------
    28520.7996150 solid Content in Inches.
    ---------------
    282)28520.799(101.137 Gallons.
        ---------
          320
           387
           1059
            2139
           -----
             165
```

Note, That 282 solid Inches is an Ale or Beer Gallon and 231 a Wine Gallon.

And 359.05 is the Square of the Diameter of a Circle that will hold a Gallon of Ale at an Inch deep, and 294.12 for Wine.

And 217.6 Inches is an *Irish* Gallon, of either Ale or Wine. Also 277.05 is the Square of the Diameter of a Circle, that will hold an *Irish* Gallon at an Inch Deep.

You may find the Content in Gallons, thus: Divide the Square of the Diameter by 359.05, and multiply the Quotient by the Depth.

359.05)1383.38 . . . (3.853
52.5

306230
18990 19265
1037 7706
19265

The Content of the greater 202.2825 Gallons.

Quest. 18. If the Diameter of a Cask at the Bung be 32 Inches, and at the Head 25 Inches, and the Length 40 Inches, how many Ale-Gallons is contain'd therein?

25	32	
25	32	359
		3
125	64	
50	96	1077
625	1024 Square of the Bung Diameter.	
	1024 the same.	
	625 Square of the Head Diameter.	

1077)2673 (2.48
40

5190
8820 99.20

204

But for *Irish* Gallons,
Divide by 831.15
The Quotient is 3.21
Which multip. by 40

Give 128.40
Irish Gallons.

Answer, 99.2 Gallons.

Otherwise, You may find a mean Diameter, and work by Scale and Compasses, thus; subtract 25 from 32, and there remains 7, which multiply'd by .7, the Product is 4.9, which added to 25, the Sum is 29.9. Then extend the Compasses from 18.95 the Gage point *English*, to 29.9, that Extent, turn'd twice from 40, (the Length) will fall upon 99.6 Gallons; something more than before.

Quest.

Quest. 19. There is a Stone 20 Inches long, 15 Inches broad, and 8 Inches thick, which weighs 217 Pounds; I demand the Length, Breadth, and Thickness of another of the same Kind and Shape, which weighs 1000 Pounds?

The Cube of 20 (the Length) is 8000. Then, (by *Eucl.* 11 33.)

As 217 : 8000 : : 1000 : 36870.645, whose Cube Root is 33.28 Inches, the Length of the Stone weighing 1000 Pounds. Then say.

As 20 : 33.28 : : 15 : 24.96
As 20 : 33.28 : : 8 : 13.312

$$\text{Answer, } \left\{ \begin{array}{ll} \text{The Length——} & 33.28 \\ \text{The Breadth——} & 24.96 \\ \text{The Thickness.} & 13.312 \end{array} \right\} \text{Inches.}$$

Quest. 20. If an Iron-Bullet, whose Diameter is 4 Inches weighs 9 Pounds, what will be the Weight of another Bullet (of the same Metal) whose Diameter is 9 Inches?

The Cube of 4 is 64, and the Cube of 9 is 729 : Then, (by *Eucl.* 12, 18.)

lb. *lb.*
As 64 : 9 : : 729 : 102.515

lb. *oz.* *dr.*
Answer, It weighs 102 8 4 *feré.*

Quest. 21. There is a square Pyramid of Marble, each Side of its Base is 5 Inches, and the Height thereof 15 Inches, and its Weight is 12 Pounds and a Quarter, I demand the Weight of another like Square Pyramid, each Side of whose Base is 30 Inches?

The Cube of 5 is 125, and the Cube of 30 is 27000. Then, (by *Eucl.* 12. 12.)

lb. *lb.*
As 125 : 12.25 : : 27000 : 2646.

Answer, The Weight is 2646 Pounds.

Quest.

Quest. 22. There is a Ball, or Globe of Marble, whose Diameter is 6 Inches, and its Weight 11 Pounds; what will be the Diameter of another Globe of the same Marble, that weighs 500 Pounds?

The Cube of 6 is 216. Then,

	lb.			lb.
As	11 :	216 : :	500 :	9818.1818

Whose Cube Root is 21.4 Inches. the Diameter sought.

Quest, 23. There is a Frustrum of a Pyramid, whose Bases are regular Octagons; each Side of the greater Base is 21 Inches and each Side of the lesser Base is 9 Inches, and its Length is 15 Feet; I demand how many solid Feet are contain'd therein?

4.8284 the tabular Number, Page 87.
237 the Square of a mean Side.

337988	21	12
144852	9	12
96568		
	189	3)144
1144.3308	48	
15		48
	237	
57216540		
11443308		

144)17164.9620(119.2

276
1324
289
9

Answer, 119.2 solid Feet.

Quest.

Quest. 24. There is a Frustrum of a Cone, the Diameter of the greater Base is 36 Inches, and the Diameter of the lesser Base is 20 Inches, and the Length or Height is 215 Inches; I demand the Length and solid Content of the whole Cone, and also the solid Content of the given Frustrum?

First, Find the Length of the whole Cone, thus;

From 36
subtr. 20

———

As 16 : 215 : : 36 : 483.75

So the Length of the whole Cone is 483 $\frac{3}{4}$ Inches.

Then find the Content of the whole Cone.

	36	1017.8784
	36	52.161
	———	———
	216	10178784
	108	6107270
	———	101788
	1296	20357
	.7854	5089
	———	——— Feet.
	5184	1728)164132.88(94.98
	6480	———
	10368	8612
	9072	17008
	———	14568
Area Base	1017.8784	———
		744

Thus I find the Solidity of the whole Cone 94.98 Feet.

Then find the solid Content of the top Part that is wanting.

.7154

```
        .7854   the Area of Unity.
          400 the Square of 20.
     ----------
    3)314.1600 Area of the lesser Base.
     ----------
       104.72  a third Part.
       268.75  Altitude of the top Part.
     ----------
        52360
       73304
      83776
     62832
    20944
    ----------
1728) 28143.5000(16.28 Feet.
     ---------  ---
    10863
       4955
       14990
     ----------
        1166
                                      Feet.
            Content of the whole      94.98
            Content of the top Piece  16.28
                                     --------
            Content of the Frustum    78.7
```

Quest. 25. If the top Part of a Cone contains 26171 solid Inches, and 200 Inches its Length, and the lower Frustrum thereof contains 159610 solid Inches; I demand the Length of the whole Cone, and the Diameter of each Base?

```
    200                         159610 }
      200                        26171 } add.
    --------                  ----------
    40000                       185781 the Sum.
        200
    ---------
    8000000
```

As

As 26171 : 8000000 : : 185781 : to 56789881, whose Cube Root is 384.3 Inches, the Length of the whole Cone.

Then find the Diameter of the lesser Base, thus;

```
200)26171
    -------
    130.855
          3
   --------
    392.565 Area of the lesser Base.
    Then, by Prob. 10, Sect. IX, Chap. I.
  As 1 : 1.2732 :: 392.565
                    1.2732
                  --------
                    785130
                  1177695
                 2747955
                 785130
                392565
                ---------
                  . . .
                  499.8137580(22.35
                  4
                  -----
                42)99
                   84
                  -----
              443)1581
                  1329
                  -----
             4465) 25237
                   22325
                   -----
                    2912
```

	Lesser Leng.		Less. Diam.		Greater Leng.		Gr. Diam
Again, as	200	:	22.35	::	384.3	:	42.94

		Inches
Answer,	The Length of the whole Cone	384.3
	The Diameter of the greater Base	42.94
	The Diameter of the lesser Base	22.35

Quest.

Quest. 26. There is a Frustrum of a Cone, whose solid Content is 20 Feet, and its Length 12 Feet; and the greater Diameter bears such Proportion to the lesser as 5 to 2; I demand the Diameters?

$5 \times 5 = 25$
$2 \times 2 = 4$
$5 \times 2 = 10$

The Sum 39

3)12

4)20(5 Feet.

These 5 Feet are the Triple of a mean Area.

Then, as 1 : 1.27324 : : 5 : 6.3662.

So the triple Square of a mean Diameter is 6.3662.

Then, as 39 : 6.3662 : : 25 : 4.080897.

This 4.080897 is the Square of the greater Diameter, whose square root is 2.020123 Feet, which is 24.24147 Inches. Then,

As 5 : 24.24147 : : 2 : 9.69659

So the greater Diameter is 24.24147, and the lesser Diameter is 9.69659 Inches.

Quest. 27. There is a Room of Wainscot 129 Feet 6 Inches in Circumference, and 16 Feet 9 Inches high, (being girt over the Mouldings;) there are two Windows, each 7 Feet 3 Inches high, and the Breadth of each, from Cheek to Cheek, 5 Feet 6 Inches; the Breadth of the Shutters of each is 4 Feet 6 Inches; the Cheek-Boards and Top and Bottom Boards of each Window, taken together, is 24 Feet 6 Inches, and their Breadth 1 Foot 9 Inches; the Door-Case 7 Feet high, and 3 Feet 6 Inches wide; the Door 3 Feet 3 Inches wide; I demand how many Yards of Wainscot are contain'd in that Room?

F.	I.	
129	6	
16	9	
782	0	
129		
64	9	
32	4	6
2169	1	6

F.	I.		
7	3		
4	6		
29	0		
3	7	6	
32	7	6	
16	3	9	Half.
48	11	3	
		2	
97	10	6	

F.	I.		
3	3		
7	0		
22	9		
11	4	6	Half.
34	1	6	

F.	I.	
24	6	
1	9	
24	6	
18	4	6
42	10	6
		2
85	9	0

F.	I.	
7	3	
5	6	
36	3	
3	7	6
39	10	6
		2
79	9	0

F.	I.	
3	6	
7	0	
24	6	add.
79	9	
104	3	

The Content of the Room	2169	1	6
The Shutters, at Work and half	97	10	6
The Door, at Work and half	34	1	6
The Cheek-Boards, &c.	85	9	0
The Sum	2386	10	6
The Window-Lights and Door-Case deduct	104	3	0
	9)2282	7	6
	253	5	

Answer, 253 Yards 5 Feet.

Quest. 28. There is a Wall which contains 18225 Cube-Feet, and the Height is 5 times the Breadth, and the Length 8 times the Height; What is the Length, Breadth, and Height?

Suppose the Breadth 2, then the Height must be 10, and the Length 80; which three Numbers multiply'd together, the Product will 1600, and the Cube of 2 is 8; then say,

As 1600 : 8 : : 18225 : 91.125.

Then the Cube Root of 91.125 is 4.5 which is the Breadth; then 5 times 4.5 is 22.5, the Height; and 8 times 22.5 is 180, the Length.

Quest. 29. There is a May-pole, whose Top-End was broken off by a Blast of Wind, and the Top-End in falling struck the Ground at 15 Feet Distance from the Foot of the May-pole, the broken Piece was 39 Feet; Now I demand the Length of the May-pole?

By *Eucl.* 1. 47. the Square of the Hypothenuse of a right-angled Triangle, is equal to the Sum of the Squares of the Base and Perpendicular.

Therefore, from the Square of 39 subtract the Square of 15; the square Root of the Remainder is the Piece standing, to which add the Piece broken off, and you have the whole Length.

39	15
39	15
351	75
117	15
1521	225
225	
1296(36	
9	
66)396	
396	

The Piece standing is 36 Feet.
The Piece broken off is 39 Feet.

The whole Length 75

Question 30.

A May-pole there was, whose Height I would know;
The Sun shining clear, strait to work I did go:
The Length of the Shadow, upon level Ground,
Just sixty five Feet, when measur'd, I found:
A Staff I had there, just five Feet in Length;
The Length of its Shadow was four Feet one Tenth:
How high was the May-pole, I gladly would know?
And it is the Thing you're desir'd to show.

By *Eucl.* 6. 4.

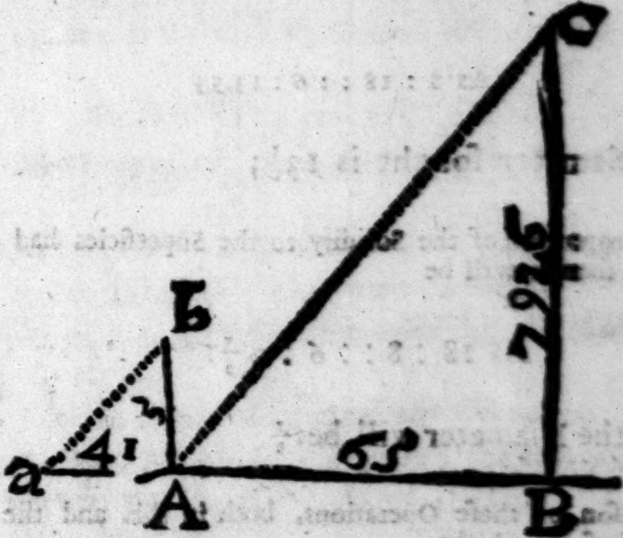
As aA : Ab : ; AB : BC.

That

That is

As $4.1 : 5 :: 65 : 79.26$;

So I find the Height of the May-pole to be 79 Feet, and a little above three Inches.



Here AB represents the Length of the Shadow of the May-pole, and BC the May-pole; aA the Shadow of the Staff, and Ab the Staff.

Quest. 31, What will be the Diameter of a Globe, when the Solidity and superficial Content thereof are equal?

If the Diameter be 1, the Solidity will be .5236, and the Superficies will be 3.1416; that is, as 1 to 6. And to find the Superficial Content, we must multiply 3.1416 by the Square of the Axis, or Diameter, and the Product is the superficial Content. And for the Solidity, multiply .5236 by the Cube of the Axis, the Product is the solid Content: Therefore because .5236 is a sixth Part of 3.1416, we must take 6 for the Diameter sought. For if 3.1416 be multiply'd by the square of 6, *viz.* by 36, the Product will be 113.0976, and if .5236 be multiply'd by the Cube of 6, *viz.* by 216, the Product is likewise 113.0976 the Solidity equal to the Superficies.

Therefore, 6 is the true Answer.

Quest. 31. What will the Axis of a Globe be, when the Solidity is in Proportion to the Superficies, as 18 to 8?

Because the Solidity and Superficies is as 1 to 6; when the Axis of the Globe is 1, it will be

As 8 : 18 : : 6 : 13.5;

So the Diameter sought is $13\frac{1}{2}$;

If the Proportion of the Solidity to the Superficies had been as 8 to 18, then it will be

As 18 : 8 : : 6 : $2\frac{1}{3}$:

So then the Diameter will be $2\frac{1}{3}$.

The Reason of these Operations, both in this and the last Question, is from Algebra.

Quest. 33. There are three Grenado Shells, of such Capacity, that the second Shell will just lye in the Concavity of the first, and the third in the Concavity of the second. The Solidity of the Metal of the first Shell is equal to its Concavity; and the Solidity of the Metal of the second, to the Concavity, is as 7 to 5; and the Solidity of the third, or least Shell's Metal, to its Concavity, is as 9 to 4. Now, supposing the Diameter of the first, or greatest Shell, to be 16 Inches, and allowing every solid Inch of Iron to weigh 4 Ounces. I demand the Diameter of the two lesser Shells, and the Thickness and Solidity of Metal of every Shell, and also the Weight of every Shell?

The Cube of 16 is 4096; then,

As 1 : .5236 : : 4096 : 2144. 6656;

the half thereof is 1072.3328, which is the Solidity of the Metal of the greater Shell, as also of the Concavity,

As .5236 : 1 : : 1072.3328 : 2048.

The Cube Root of 2048 is 12.699, which is the Diameter of the second Shell.

The Sum of 7 and 5 is 12; then,

As 12 : 5 : : 1072.3328 : 446.805.

This 446.805 is the solid Content of the Concavity of the second.

As .5236 : 1 : : 446. 805 : 853.333

The Cube Root of 853.333 is 9.485, the Diameter of the least Shell.

The Sum of 9 and 4 is 13. then,

As 13 : 4 : : 446.805 : 137.47846.

This 137.47846. is the solid Content of the Concavity of the third.

As .5236 : 1 : : 137.47846 : 262.5639.

The Cube Root of 262.5639 is 6.4034, the Diameter of the least Shell's Concavity.

From 16. the Diameter of the greatest,
subtr. 12.699 the Diameter of the Second.

Rem. 3.301

half is = 1.65 the Thickness of Metal of the greatest.

From 12.699 the Diameter of the second,
subtr. 9.485 the Diameter of the least.

Rem. 3.214

half is = 1.697 the Thickness of Metal of the second.
From 9.485 the Diameter of the least.
subtr. 6.403 the Diameter of the Concavity.

Rem. 3.082

half is = 1.541 the Thickness of Metal of the least.

The

The Metal of the greatest is 1072.33 solid Inches; which divide by 4. (because every solid Inch is a Quarter of a Pound) the Quotient is 268.08 Pounds.

The metal of the second is 625.52 solid Inches; which divided by 4, the Quotient is 156.38 Pounds, the Weight of the second.

The Metal of the least Shell is 309.32 solid Inches; which divided by 4, the Quotient is 77.33 Pounds, the Weight of the least.

The Diam. of the { second Shell 12.699 / least Shell — 9.485 } Inches.

The Thickness of the Metal, of the { greatest 1.65 / second 1.607 / least — 1.541 } Inches.

The Weight of the { greatest 268.08 / second 156.38 / least — 77.33 } Pounds.

A SHORT APPENDIX.

§ I. *Of* Gaging, *according to both the* Irish *and* English *Gallon.*

I Shall not here give the whole Art of Gaging, (there being several Books of that Art already in Print, writ by better Hands) but shall only lay down some short Practical Rules, whereby any Artificer, or others, may find the Quantity of Liquor in any Vessel upon Occasion.

Problem I.

To find the several Multipliers, Divisors, and Gage-points belonging to the several Measures now used in England and Ireland.

282)1.0000(.003546 Multiplier for Ale Gallons.
231)1.0000(.004329 Multiplier for Wine Gallons.

268.8)1.000(.0037202 Multiplier for Corn Gallons.
2150.42)1.000(.00046502 Multiplier for Corn Bushels.
217.6)1.0000(.0045955 Multiplier for *Irish* Gallons.

So, if the solid Inches in any Vessel be multiply'd by the said Multipliers, the Product will be Gallons in the respective Measures; or, dividing by the Divisors 282, 231.268.8, or 217.6 the Quotients will likewise be Gallons.

Note, That 282 solid Inches is a Gallon of Ale, or Beer-Measure; 231 solid Inches is a Gallon of Wine-Measure; 268.8 solid Inches is a Gallon, and 2150.42 solid Inches is a Bushel of Corn-Measure. Also 217.6 solid Inches, is a Gallon *Irish*, both of Ale, Wine, or Oyl.

For circular Area's, the following Multipliers and Divisors are to be used.

282).785398(.002785 Multiplier for Ale Gallons.
231).785398(.003399 Multiplier for Wine Gall.
217.6).785398(.0036093 Multip. for *Irish* Gall.
.785398)282.(359.05 Divisor for Ale Gall.
.785398)231.(294.12 Divisor for Wine Gall.
.785398)2150.42(2738 Divisor for Corn Bushels.
.785398)217.6(277.05 Divisor for *Irish* Gallons.

The Gage-Point for Squares in	Ale-Measure, is	16.79
	Wine-Measure, is	15.19
	Malt-Bushel, is	46.36
	Irish-Gallon, is	14.75

The Gage-Point for circular Figures in	Ale-Measure,	18.95
	Wine-Measure, is	17.15
	Malt-Bushel, is	52.32
	Irish-Gallon, is	16.64

PROB-

PROBLEM II.

To find the Area in Ale, Wine, or Irish *Gallons, of any rectilineal plane Figure, whether Triangular, Quadrangular, or Multangular.*

TO resolve this Problem, you must, by Chap. I, Part II, find the Area in Inches by the proper Divisor, *viz.* by 282 for Ale, or by 231 for Wine, or 217.6 for *Irish*, or else by Multiplication, by .003546 for Ale, by .004329 for Wine; by .004595 for *Irish*, and the Quotient or Product will be the Area,

Example. Suppose a Back or Cooler in the Form of a Parallelogram, or Long-Square, 252 Inches in Length, and 84.5 Inches in Breadth, what is the Area in Ale, Wine, or *Irish* Gallons.

Multiply 252 by 34.5, and the Product is 21125, the Area in Inches, which divided by 282, and the Quotient is 74.9 Gallons of Ale; or multiply'd by .003546, the Product is 74.90925 Gallons, nearly the same; and if 21125 be divided by 231, or multiply'd by .004329 it will give 91.45 Gallons of Wine. And if 21125 be divided by 217.6, or Multiplied by .004595 it will give 97.06 *Irish* Gallons.

By Scale *and* Compasses.

Extend the Compasses from 282 to 250, that Extent will reach from 84.5 to 74.9: And,

Extend from 231 to 250, that Extent will reach from 84.5 to 91.45.

Extend the Compasses from 217.6 to 250, that Extent will reach from 84.5 to 97.06.

NOTE,

NOTE, The Area's of all Superficies are always to be understood to be 1 Inch Deep, otherwise, it could not be said that the Area of such a Parallelogram, Circle, *&c.* is so many Gallons.

Having found the Area of a Back or Cooler, the next Thing will be to find out the true Dipping or Gaging-Place in that Back, that so the true Quantity of Worts may be computed at any Depth, which may be thus done.

1. When the Bottom of the Back is cover'd all over (of any Depth) with Worts, or other Liquor then dip it in eight or ten several Places, (more or less, according to the Largeness of the Back) as remote and equally distant from each other as you can well do, noting down the wet Inches and decimal Parts of every Dip.

2. Divide the Sum of all those Dips by the Number of Places you dip'd in, and the Quotient will be the mean Wet of all those Dips.

3. Lastly, find out such a Place by the Side of the Back (if you can) that just wets the same with that mean Dip, and make a Notch or Mark there for the true and constant Dipping-Place of that Back;

Then, if any Quantity of Worts (which covers the whole Back) be dipped or gaged at that Place, and the Wet Inches so taken, be multiply'd into the Area of the Back in Gallons, the Product will shew how many Gallons of Wort are in that Back at that Time, provided the Sides of the Back do stand at Right-angles with the Bottom.

PROBLEM III.

The Diameter of a Circle being given in Inches, to find the Area thereof in Ale, Wine, or Irish *Gallons.*

IF the Square of the Diameter be multiply'd by .002785 for Ale, or by .003399 for Wine, or by .0036093 for *Irish* Gallons, or if it be divided by 359.05 for Ale, or by 294.12 for

for Wine, or by 277.05 for *Irish* Gallons, the Products or Quotients will be the respective Ale, Wine, or *Irish* Gallons.

Example. Suppose the Diameter of a Circle be 32.6 Inches, what will the Area in Ale, Wine, or *Irish* Gallons?

The Square of 32.6 is 1062.76.

Then 359.05) 1062.76(2.9599 Area in Ale Gall.
And 294.12) 1062.76(3.6133 Area in Wine Gall.
And 277.05) 1062.76(3.8565 Area in *Irish* Gall.
Or 1062.76x.002785 = 2.9599 Ale Gall.
And 1062.76x.003399 = 3.6133 Wine Gall.
And 1062.76x.003609 = 30565 *Irish* Gall.

By Scale *and* Compasses.

Extend the Compasses from 18.95 (the Gage-Point for Ale) to 32.6 (the Diameter) that Extent will reach from 1 to a 4th Number, and from that 4th to 2.9599 Gallons. Or, extend the Compasses from 1 to 32.6, that Extent turn'd twice over from .002785, will at last fall upon 2.9599.

For Wine, extend from 17.15, (the Gage-Point for Wine) to 32.6 (the Diameter) that Extent turn'd twice over from 1, will at last fall upon 3.6133 Gallons

Or thus: Extend from 1 to 32.6, that Extent will reach from .003399 (being turn'd twice over) to 3.6133 Wine Gallons.

For *Irish* Gallons, Extend from 16.64 (the *Irish* Gage-point) to 32.6 (the Diameter) that extent turn'd twice over from 1 will at last fall upon 3.8565 the *Irish* Gallons sought.

PROB-

Problem IV.

The Transverse (or Longest Diameter) and the Conjugate (or shortest Diameter) of an Ellipsis (or Oval) being given, to find its Area in Ale, Wine, or Irish Gallons.

If the Rectangle, or Product of the two Diameters, that is of the Length and Breadth of the Oval, be divided by 359.35, or multiply'd by .002785 for Ale, or divided by 294.12, or multiplied by .003399 for Wine, or divided by 277.05, or multiplied by .0036093 the Quotients or Products will be the Ale Wine or *Irish* Gallons requir'd.

Example. Suppose the longest Diameter be 81.4 Inches, and the shortest Diameter be 54.6 Inches, what will be the Area of that Oval?

Multiply 81.4 by 54.6, and the Product is 4444.44; then

359.05)4444.44(12.38 Area in Ale-Gallons.
294.12)4444.44(15.11 Area in Wine-Gallons.
277.05)4444.44(16.71 Area in *Irish* Gallons.
Or 4444.44x.002785 = 12.38 Ale Gallons.
And 4444.44x.003399 = 15.18 Wine Gallons.
And 4444.44x.003609 = 16.1 *Irish* Gallons.

By Scale *and* Compasses.

First, find a mean Proportional between 81.4 and 54.6. by dividing the Distance between them into two equal Parts, and the middle Point will be at 66.6, which is the mean Proportional (that is, the Diameter of a Circle equal to the Oval) Then extend the Compasses from 18.95 (the Gage-point for Ale) to 66.6, that Extent turn'd twice over from 1, will at last fall upon 12.38.

Alt-

Ale-Gallons: And extend from 17.15 (the Gage-point for Wine) to 66.6; that Extent turn'd twice over from 1, will reach at last to 15.11 Wine-Gallons.

Lastly, Extend the Compasses from 16.64 (the *Irish* Gage-point) to 66.6 that extent turn'd twice over from 1 will at last fall upon 16.1 *Irish* Gallons.

Problem V.

To find the Content in Ale, Wine, or Irish Gallons of any Prism, what Form soever its Base is of.

FIRST find its solid Content in Inches (by Sect. 1, 2, 3, of Chap. II. Part II.) then divide that Content in Inches by 282 for Ale; or by 231 for Wine, or by 217.6 for *Irish*, the respective Quotients will be the Content in Wine, Ale, or *Irish* Gallons.

Otherwise, you may find the Content of a Prism by finding the Area of its Base in Gallons, (by Problem II. of this Appendix) and multiply that Area by the Tun's Height, or Depth within, the Product will be its Content in Gallons.

Example. Suppose a Tun, whose Base is a Parallelogram right-angled, its Length being 49.3 Inches, its Breadth 36.5 Inches, and the Depth of the Tun is 42.6 Inches; the Content in Ale Wine and *Irish* Gallons is requir'd.

The Length, Breadth, and Depth, being multiply'd continually, the Product is 76656.57; which divided by 282, the Quotient is 271.83 Ale-Gallons: And divided by 231, the Quotient is 331.84 Wine Gallons. And divided by 217.6, the Quotient is 352.23 *Irish* Gallons. And by dividing by 2150.4, such a Cistern will be found to hold 35.65 Bushels of Corn.

B

By Scale *and* Compasses.

Extend the Compasses from 282 to 36.5, (the Breadth of the Base) that Extent will reach from 49.3 (its Length) to 6.38 Ale-Gallons, the Area of the Base; then extend from 1 to 42.6, (the Depth) that Extent will reach from 6.41 (the Area of the Base to 271.8 Gallons (the Content.)

Problem VI.

To find the Content of a Tun, whose Bases are alike and Parallel, but unequal, being the Frustrum of a Pyramid.

FIND the Area of each Base, and a mean Proportional between them, and multiply the Sum of those three by one third Part of the Depth or Height, and the Product is the Content.

Example. Suppose a Tun, whose Bases are Parallellograms, the Length of the greater is 100 Inches, and its Breadth 70 Inches; the Length of the lesser Base 80, and its Breadth 56; and the Depth of the Tun 42 Inches, the Content in Ale and Wine Gallons is requir'd.

Multiply 100 by 70, the Product is 7000, the Area of the greater Base; and 80 multiplied by 56, the Product is 4480, the Area of the lesser Base; then multiply the two Area's into each other, and the Product is 31360000, whose Square Root is 5600, a geometrical mean Proportional.

The

The greater Area	7000	add
The lesser Area	4480	
The mean Proportional	5600	
	17080	
A third of the Depth	14	
	68320	
	17080	

282)239120(847.94 A. G.
231)239120(1035.15 W. G.
217.6)239120(1098.9 *Irish* G.

PROBLEM VII.

To find the Content of a Tun, whose Bases are Parallel and circular, being the Frustrum of a Cone.

YOU may find the Content as in the last Problem, by multiplying the Sum of the Areas of the two Bases, and a mean Proportional, by one third Part of the Depth.

But it will be a shorter Way to find the Area of a mean Circle in Gallons, and multiply that by the Depth, thus: To the Rectangle of the greater and lesser Diameters, add one third Part of the Square of the Difference of the Diameters, that Sum is the Square of a mean Diameter; which divided by 359.05 for Ale, or by 294.12 for Wine, or by 277.05 for *Irish* Gallons, gives the Area of a mean Circle in Ale, Wine or *Irish* Gallons; which multiply'd by the Depth, gives the Content.

Example. Suppose the greater Diameter 80 Inches, and the lesser Diameter 71 Inches, and the Depth 34 Inches, the Content in Ale, Wine and *Irish* Gallons is requir'd.

Multiply 80 by 71, and the Product is 5680; to which add 27, (a third Part of the Square of the Difference of the Diameters) and the Sum is 5708, which is the Square of a mean Diameter; which divide by 359.05, and the Quotient will be 15.895 Ale Gallons the Area; which multiply by 34, (the Depth) and the Product will be 540.43 Gallons, the Content.

By Scale *and* Compasses.

Add the two Diameters together, and take half the Sum, which is 75.5; which take for a mean Diameter; (tho' it is not exact, yet it will be near enough the Truth, if the Difference between the Diameters be not great) extend the Compasses from 18.95 (the Gage-point for Ale) to 75.5, (the mean Diameter) that Extent will reach from 34, (the Depth) to a 4th Number, and from that to 540.4 Gallons, the Content.

And if you extend the Compasses from 17.15 (the Gage-point for Wine) to 75.5, that Extent will reach from 34 (twice turn'd over) to 659.7 Gallons of Wine.

Lastly, If you extend the Compasses from 16.64 the *Irish* Gage-point, to 75.5 that extent will reach from 34, (twice turn'd over) to 700.3 *Irish* Gallons.

The Method used by the Gagers for all such Tuns, is to take the Diameter in the Middle of every 10 Inches; that is, at five Inches from the Bottom, and at 15, and at 25, *&c.*

Then they find the Area to every one of these Diameters, and enter them in their Books. Then, when they survey, they take the Wet Inches and Parts that the Liquor in the Tun is in Depth, and every 10 Inches they take the respective Areas, and remove the separating Point one Place towards the right Hand, and for what odd Inches of the Depth above the even Tens, they multiply the next Area by them, and so add all the several Products together, and the Total will be the Gallons of Liquor in the Tun.

Example. Suppose the Diameter at 5 Inches from the Bottom be 64 Inches, and at 15 Inches from the Bottom 67 Inches, and at 25 Inches 70 Inches, and at 35 Inches from the Bottom, the Diameter

meter is 73 Inches. Now the Area answering to 64 Inches, is 11.4078 Ale-Gallons; and to 67 Inches, is 12.5023 Gallons; and the Area to 70 Inches, is 13.647 Gallons; and to 73, is 14.8418 Gallons: Then, supposing the Depth of the Liquor in the said Tun be found to be 33.6 Inches: Now, to cast up this Gage, first, In the Area answering to 64 Inches, being multiply'd by 10, that is, by removing the separating Point a Place towards the right Hand, it will be 114.078 Gallons; and the next will be 125.023; and the next 136.47 Gallons: Now these three will be the Content to 30 Inches deep. Then, to find the Content of the 3.6 Inches, multiply the next Area 14.8418 by 3.6, and the Product is 53.4305: Add all these together, and the Sum is the whole Quantity of Liquor in the Tun.

The Content at 10 Inches deep	114.078
The Content at the next 10 Inches	125.023
The Content of the next 10 Inches	136.470
The Content of the next 3 Inches	53.430
The whole Quantity of Liquor in the Tun, Ale-Gallons.	429.001

Problem VIII.

To find the Drip or Fall of a Tun.

SUPPOSE the Tun last mention'd was so plac'd, that when the Bottom is but just cover'd on one Side, the Liquor is 4 Inches deep on the Side opposite, How much must be allow'd for the Fall of this Tun; that is, how much Liquor is there in the Tun?

The Diameter in the Middle of 4 Inches from the Bottom, is 61.6 Inches; and the Area answering thereunto, is 10.568; which multiply'd by 2, (that is, half 4) the Product is 21.136 Ale-Gallons; and so much Liquor will just cover the Bottom.

But suppose it was set so much on one Side, as to be 30 Inches deep on one Side, when the Liquor on the opposite Side just cuts between the Bottom and Staves, How much Liquor will there be in the Tun?

Square the bottom Diameter, and multiply that Square by the top Diameter, and divide the last Product by the Sum of the Diameters, and to the Quotient add the Square of the bottom Diameter, and divide the Sum by 1077.15 for Ale, or by 882.36 for Wine, or by 831.15 for *Irish* Gallons, multiply the Quotient by the Depth, the Product is the Content.

The bottom Diameter of the fore-mention'd Tun, is 61 Inches; and the Diameter at 30 Inches from the Bottom, is 71.5 Inches; the Square of 61, is 3721; which multiply'd by 71.5, the Product is 266051.5; this divided by 132.5, (the Sum of the Diameters) the Quotient is 2007.936; to which add 3721, (the Square of 61(and the Sum will be 5728.936; this divided by 1077.15; and the Quotient is 5.3186; which multiply'd by 30, (the Depth) the Product is 159.558, the Gallons of Liquor in the Tun.

When the Frustrum of a Cone, or Pyramid, is cut by a Diagonal Plane thro' the Extremities of the Diameters, (as the Liquor in the Tun represents) such Solid is call'd a Hoof, (*Vide Ward's* Young Mathematician's Guide, Pag. 414.)

If it be the Hoof of a square Frustrum, instead of dividing by 1077.15, divide by 846 for Ale, or by 693 for Wine, or by 652.3 for *Irish* Gallons. All the rest of the Work is the same.

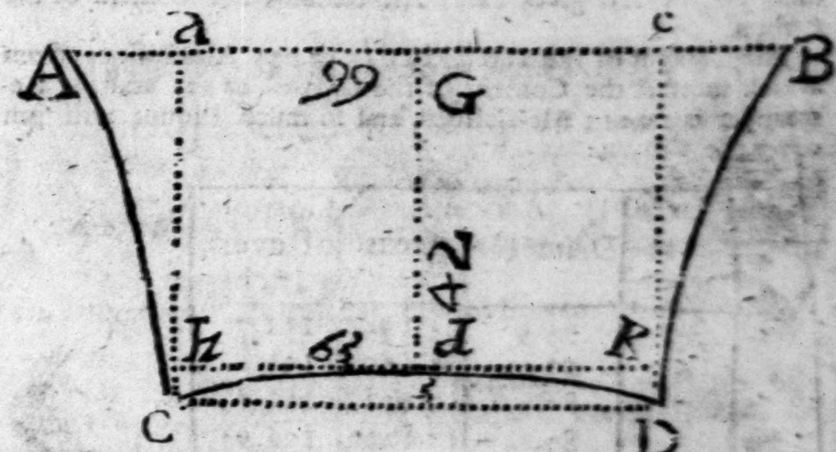
PROB-

PROBLEM IX.

To Gage a COPPER.

LET ABCD be a small Copper to be Gaged.

Take a small Cord, or Packthread; make one End fast at A, and extend the other to the opposite Side of the Copper at B, where make it fast, or cause some Person to hold it very strait; then set one End of the Instrument in the Bottom of the Copper at C, and move it to and fro, till you find the nearest Distance to the Thread (as at a): This Distance, aC, is the Depth of the Copper, which suppose to be 47 Inches.



In like manner, set the End of the Rule upon the Top of the Crown at d, and take the nearest Distance to the Thread, (as dG) which suppose 42 Inches, this subtracted from aC, 47, the Remainder 5 is the Altitude of the Crown.

To find CD, the Diameter of the Bottom of the Crown;

Measure AB, the Diameter of the Top, which, admit it be 99 Inches, then hold a Thread so as a Plummet at the End thereof may hang just over C, by which Means you will find the

the Distance Aa. Do the like on the other Side; so will you find also the Distance, cB; which suppose 17.5 Inches each, add these two together, and subtract their Sum (*viz.* 35) from 99, and the Remainder is 64 Inches, the Diameter at the Bottom of the Crown: The Diameter which touches the Top of the Crown, may be found by the Sliding-Rule to be 65 Inches.

Now, to find the Content of the Copper from the Crown upwards, (that is, the Part ABkh) the Depth gd being 42 Inches, you may take a Diameter in the *Middle* of every 6 Inches of the Depth, which suppose to be as in the second Column of the following Table, the Numbers in the third Column are the respective Areas in Ale-Gallons, found by Problem III; the fourth Column shews the Content of every 6 Inches; all which being added together, the Sum will be the Content of that Part, ABkh; that is, so much as it will hold after the Crown is covered.

Now, if the Crown be taken for the Frustrum of a Sphere, the Content (by the latter Part of Sect. II. Pag. 179) will be found to be 28.75 Ale-Gallons.

But may be more readily found, very near the Truth, thus:

The Diameter, CD, was found to be 64, and the Area to this Diameter is 11.408; this multiply'd by half the Crown's Altitude, *viz.* by 2.5, gives 28.52 Ale-Gallons, the Content of the Crown.

The Content of the Part hkDC, is 57.935 Ale-Gallons; from which subtract the Content of the Crown, 28.52, and the Remainder is 29.415 Ale-Gallons, and so much Liquor will just cover the Crown.

Parts of the Depth	Diameter	Areas.	Content of every 6 Inches
6	95.3	25.2945	151.767
6	90.0	22.6095	135.657
6	85.0	20.1223	120.734
6	80.	17.8246	106.947
6	75.2	15.7499	94.499
6	70.5	13.8426	83.056
6	66.	12.1319	72.791
The Sum			765.451
To just cover the Crown			29.415
The whole Content			794.866 Ale-Gallons.

By

By Scale and Compasses.

You may find the Areas answering to every one of the Diameters, thus;

Extend the Compasses from the Gage-point to the Diameter, that Extent being turn'd twice over from 1, will at last fall upon the Area of that Circle: Or, being turn'd twice over from 6, will give the Content of that 6 Inches of the Depth.

Example. Extend the Compasses from 18.95 (the Gage-point) to 95.3, that Extent turn'd twice over from 6, will at last fall upon 151.76 Gallons, the Content of the first 6 Inches. And so of the rest.

PROBLEM X.

To compute the Content of any Close Cask.

IN order to perform this difficult Part of Gaging, the three following Dimensions of the Cask must be truly taken,

Viz. The Bung-Diameter, The Head-Diameter, The Length of the Cask, } within the Cask.

In taking these Dimensions, it must be carefully observ'd,

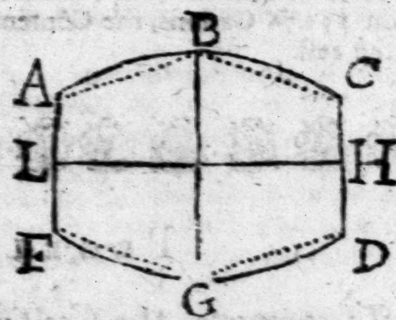
1. That the Bung-hole be in the Middle of the Cask; also, that the Bung-staff and the Staff opposite to the Bung-hole, are both regular, and even within.

2. That the Heads of the Cask are equal, and truly circular; if so, the Distance between the Inside of the Chine to the Outside of its opposite Staff, will be the Head-Diameter within the Cask, very near.

3. With a sliding Pair of Calipers (made for that Use) take the shortest Distance, or Length, between the Outsides of the two

two Heads; from that Length subtract 1 $\frac{1}{4}$ Inch (more or less, according to the Largeness of the Cask) for the Thickness of the Head: The Remainder will be the Length of the Cask within. But if the Cask be empty, you may take the Length, by putting a strait Rod in at the Tap-hole, and allow for the Thickness of the Head.

Now, by these Dimensions, one would think the Content of the Cask was perfectly limited, but it will be easy to perceive, by the following Figure, that the Diameters and Length of one Cask may be equal to those of another, and yet one of those Casks may contain several Gallons more than the other.



As for Instance, the Figure ABCDF is suppos'd to represent a Cask: Then it is plain, that if the outward curv'd Lines, ABC, and FGD, are the Bounds, or Staves of the Cask, it must needs hold more than if the inner prick'd Lines were the Bounds, or Staves; and yet the Bung-Diameter BG, and Head-Diameters CD and AF, and, Length LH, are the same in both those Casks.

Whence it appears, that no one general Rule can be given whereby the Content of all Sorts of Casks can be gaged. And therefore Gagers do usually suppose every Cask to be in some of these Forms.

1. The middle Frustrum of a Spheroid.
2. The middle Frustrum of a Parabolick Spindle.
3. The lower Frustrums of two equal Parabolick Conoids.
4. The lower Frustums of two equal Cones.

2. If the Staves of the Cask be very much curved, (as the outward Lines of the last Figure) then the Cask is supposed to be the Middle Frustrum of a Spheroid.

3. If the Staves (between the Bung and Head) be something less curved, then the Cask is taken to be the Middle Frustrum of a Parabolick Spindle

If

3. If the Staves (between the Bung and Head) be very little curved, then the Cask is taken to be the lower Frustrums of two equal Parabolick Conoids, abutting, or joining together, upon one common Base.

4. If the Staves (between the Bung and Head) be strait, as the prick'd Lines in the last Figure) then the Cask is taken to be the lower Frustrums of two equal Cones, abutting or joining together upon one common Base.

There are several Rules laid down in Books of Gaging, for finding the Content of each several Form, but I think the shortest and most practical Way, is to find such a mean Diameter, which will reduce the propos'd Cask to a Cylinder. Thus,

Multiply the Difference of the Bung and Head-Diameters by .7 for a Spheroid; by .65 for the second Form, by .6 for the third Form, and by .55 for the fourth Form; and add the Product to the Head-Diameter, and the Sum is a mean Diameter

Example. Suppose the Bung-Diameter be 32 Inches, the Head-Diameter 24 Inches, and the Length 40 Inches, the Content in each Variety is requir'd.

The Difference between the Bung and Head-Diameters is 8; which multiply'd by .7, the Product is 5.6; which added to the Head-Diameter, the Sum is 29.6, the mean Diameter: The Area answering thereunto, will be found (by Prob. III.) to be 2.44 Ale-Gallons; which multiply'd by the Length, the Product is 97.4 Gallons; and so much is the Content, if it be the first Form.

Again, if the Difference of the Diameters 8 be multiply'd by .65, the Product will be 5.2; which added to the Head-Diameter, the Sum is 29.2, for the mean Diameter; and the Area answering thereunto, is 2.3746 Gallons; which multiply'd by 40, (the Length) the Product is 94.98 Gallons, the Content, if it be of the second Form.

Again, If the Difference 8 be multiply'd by .6, the Product is 4.8; which added to the Head-Diameter, the Sum is 28.8, the mean Diameter; the Area thereunto is 2.31 Gallons; which multiply'd by 40, gives the Content 92.4 Gallons, for the third Form.

Again, the Difference 8, multiply'd by .55, the Product is 4.4; which added to the Head-Diameter, makes the mean Diameter, 28.4, the Area thereof is 2.2463; which multiply'd by 40, the Product is 89.85 Gallons, for the third Form.

By

By Scale *and* Compasses.

Extend the Compasses from the Gage-point 18.95, to the first mean Diameter 29.6, that Extent will reach from the Length, 40, to a fourth Number, and then to the Content, 97.4 Ale-Gallons.

Again, Extend from 18.95 to 29.2, (the second mean Diameter) that Extent turn'd twice over from 40, will at last fall upon 94.98 Gallons.

Again, Extend from 18.95 to 28.8, (the third mean Diameter) that Extent, turn'd twice over from 40, will at last fall upon 92.4 Gallons.

Again, Extend from 18.95 to 28.4 (the fourth mean Diameter) that Extent, turn'd twice over from 40, will at last fall upon 89.85 Gallons.

Altho I have all along made Use of the Line of Numbers upon the common Two-Foot, or Eighteen-Inch Rules, for the Reason mention'd in the *Preface*; yet the Rules may easily be apply'd to the Sliding-Rule, thus: To find the Area of a Circle in Gallons, set the Gage-point upon D, (that is, a single Line of Numbers) to 1 upon C, (that is, a double Line) then against any Diameter upon D, is the Area upon C, thus:

To find the Content of the Cask, last mention'd, the first Form,

Set the Gage-point 18.95 upon D, to the Length 40 upon C; then, against the mean Diameter 29.6 upon D, is 97.4 Gallons, Content upon C.

And against 29.2 (the next mean Diameter) on D, is 94.98 Gallons on C.

And against 28.8 (the next mean Diameter) on D, is 92.4 Gallons on C.

And against 28.4 the last mean Diameter) on D, is 89.85 Gallons on C.

All done without removing the Slider.

A TABLE

A TABLE *of the Segments of a Circle whose Area is Unity.*

V. S.	Seg^t	V. S.	Seg^t	V. S	Seg^t	V. S.	Seg^t
1	.0017	99	.9983	26	.2066	74	7934
2	.0048	98	9952	27	.2178	73	7822
3	.0087	97	.9913	28	.2292	72	.7708
4	.0134	96	.9866	29	.2407	71	.7593
5	.0187	95	.9813	30	.2523	70	.7477
6	.0245	94	.9755	31	.2640	69	7360
7	.0308	93	.9692	32	.2759	68	.7241
8	.0375	92	.9625	33	.2878	67	.7122
9	.0446	91	.9554	34	.2998	66	.7002
10	.0520	90	.9480	35	.3119	65	.6881
11	.0598	89	.9402	36	.3241	64	.6759
12	.0680	88	.9320	37	.3364	63	6636
13	.0764	87	.9236	38	.3487	62	.6513
14	.0851	86	.9145	39	.3611	61	.6389
15	.0941	85	.905	40	.3739	60	.6265
16	.1033	84	.8967	41	3860	59	.6140
17	.1127	83	.8873	42	.3986	58	.6014
18	.122[illegible]	82	.8776	43	4112	57	.5888
19	.1323	81	.8677	44	4238	56	.5762
20	.1424	80	.8576	45	.4364	55	.5636
21	.1526	79	.8474	46	.4491	54	.5509
22	.1631	78	.8369	47	.4618	53	.5382
23	.1737	77	.8263	48	.4745	52	.5255
24	.1845	76	.8155	49	.4873	51	.5127
25	.1955	75	.8045	50	.5000	50	.5000

The

The Use of the Table of Segments.

§ 1. *Is to find the Ullage, or Quantity of Liquor remaining in a Cask, whose Axis is parallel to the Horizon, the Surface of the Liquor cutting the Heads of the Cask.*

The RULE is,

TO the Wet or Dry Inches of the Bung-Diameter add a competent Number of Cyphers; then divide it by the whole Diameter, the Quotient found in the Table under the Title V. S. gives a Segment; which multiply'd by the whole Content of the Cask, the Product shews the Quantity of Liquor in the Cask, if the Dividend was the Wet Inches; or the Ullage, if it was the Dry.

Let there be a Cask in Form of a Cylinder, whose Bung-Diameter is 29 Inches, the Dry Part 13, and the Wet 16, and the Content 80 Gallons; How many Gallons are wanting to fill the Cask?

Divide the Dry Inches, 13, by 29, the Bung-Diameter, and the Quotient is .448; find the two first Figures .44 under V. S. and the Segment against it is .4238; to which add a proportional Part for the 8, and the whole Segment will .4333; which multiply'd by the Content of the Cask, the Product will be 34.664 Gallons; and so much the Cask wants of being full.

Note, If the Cask be in the Form of a Cylinder, or near that Figure, the Table will give the Ullage exact enough; but if it be a spheroidal Cask, then use the following Method:

1. By the Bung and Head-Diameters, find such a mean Diameter as you judge will reduce the propos'd Cask to a Cylinder, and then find its Content.

2. From

2. From the Bung-Diameter subtract the mean Diameter, and take half the Difference.

3. From the Wet Inches subtract the said half Difference; reserve this Difference; then use this Proportion:

As the mean Diameter: is to 1000,
(the Diameter of the Tabular Circle) : :
so is the reserv'd Difference:
to a Versed Sine in the Table.

Then, if the Tabular Segment be multiply'd into the Content (as before,) the product will be the Quantity of Liquor in the Cask.

Example. Let the Cask be the same as in Page 300 of the first Form, where the Bung-Diameter is 32 Inches, and the mean Diameter 29.6, and the Content 97.4 Gallons; and suppose the Wet Inches 19, to find the Quantity of Liquor in the Cask.

From	32	From	19
subtr.	29.6	subtr.	1.2
Rem.	2.4	Rem.	17.8 reserv'd.
Half	1.2		

As 29.6 : 100 : : 17.8 : .60, the V. S.

The Segment to 60 is .6265, which multiply'd by 97.4 the Content, the Product is 61 Gallons, the Quantity of Liquor in the Cask.

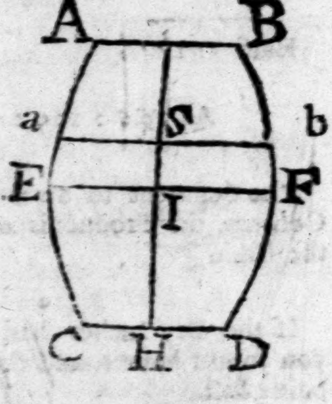
If the Dry Inches had been given, by the same Method you might have found the Ullage, or what the Cask wanted of being full.

§. 2. *To find what Quantity of Liquor is in a Cask, when its Axis is perpendicular to the Horizon,* viz. *when it stands upright upon one of its Heads.*

To do this, you must know how to calculate the Area of any Circle, between the Bung and Head, whose Distance from the Bung, or Middle of the Cask, is given; which may be done by this Proportion.

As the Square of half the Length of the Cask: is to the Difference between the Bung and Head-Areas, : : so is the Square of any Circle's Distance from the Bung: to the Difference between the Bung-Area and the Area of that Circle, *viz*., the Area of the Liquor's Surface.

Then, from the Bung-Area subtract one third Part of the aforesaid Difference, *viz.* between the Bung-Area and the Area of the Liquor's Surface? Multiply the Remainder by the Liquor's Distance from the Bung, and the Product will shew what Quantity of Liquor is either above or under half the Content of the Cask.



Example. Let us again suppose the Cask in Page 300, whose Length is 40 Inches, Bung-Diameter 32, and Head-Diameter 24, and suppose the Wet Inches, SH, 26 Inches.

The Square of half the Length is 400, the Distance of the Liquor's Surface from the Bung SI is 6, whose Square is 36; the Area of the Bung Diameter 2.8519 Ale Gallons, and the Area of the Head Diameter 1.6042, the Difference 1.2477. Then,

As 400 : 1.2477 : : 36 : .0751

One third is = .0250

From

From	2.8519	Bung-Area,
subtr.	.0250	a third of the Difference.
Rem.	2.8269	
	6	mult. Dist. from the Bung.
	169614	Content above the Bung.
Add	487	half the Content Cask.
	65.66	the Quantity of Liquor in the Cask.

PROBLEM XI.

Gaging of MALT.

TO find the Quantity of Malt in a Cistern, or upon a Floor.

First, find the Area of the Base in Bushels by multiplying the Length by the Breadth, and dividing the Product by 2150.42, or only by 2150. and multiply that Area by the mean Depth; (how to take the mean Depth, see Problem II.) If the Base be Circular, or Oval, divide by 2738. (See Problem I)

Example. There is a Cistern, whose Length is 84 Inches, and and Breadth 54 Inches, and the mean Depth is 43.6 Inches, What is the Content?

Multiply 84 by 54, and the Product is 4536; which divide by 2150, (and the Quotient is 2.1053 Bushels, the Area of the Bottom at 1 Inch deep; which multiply'd by the Depth 43.6, and the Product is 91.98 Bushels, the Content.

Example.

Example. Suppose a Quantity of Malt upon a Floor, whose Length is 245 Inches, and the Breadth 184 Inches, and the mean Depth 5,6 Inches, how many Bushels are there?

Multiply 245 by 184, and the Product is 45080; which divided by 2150, the Quotient is 20.967, the Area of the Base; which multiply'd by the mean Depth, the Product is 117.4 Bushels the Content.

By the sliding Rule.

There is an inverted Line of Numbers upon some Sliding-Rules, mark'd with the Letter M, which was contriv'd purposely for gaging of Malt; and there is a double Line of Numbers upon the Rule, and upon the Slider two double Lines of Numbers; all of these are of equal Radius, and all work together at once: Thus, set the Length and Breadth against one another upon the inverted Line, and that which slides by it; then, on the other Edge of the Rule, against the Depth, you will find the Content in Bushels. Thus, in the first Example, Set 54 upon the Slider against 84, upon the inverted Line, and then, against 43.6 upon the other Part of of the Rule, is 91.98 upon the Slider.

Again, in the second Example, set 184 upon the Slider to 245 upon the inverted Line; and against 5.6 upon the other Part of the Rule, is 117.4 upon the Slider.

§. II. *Of* LAND-MEASURING.

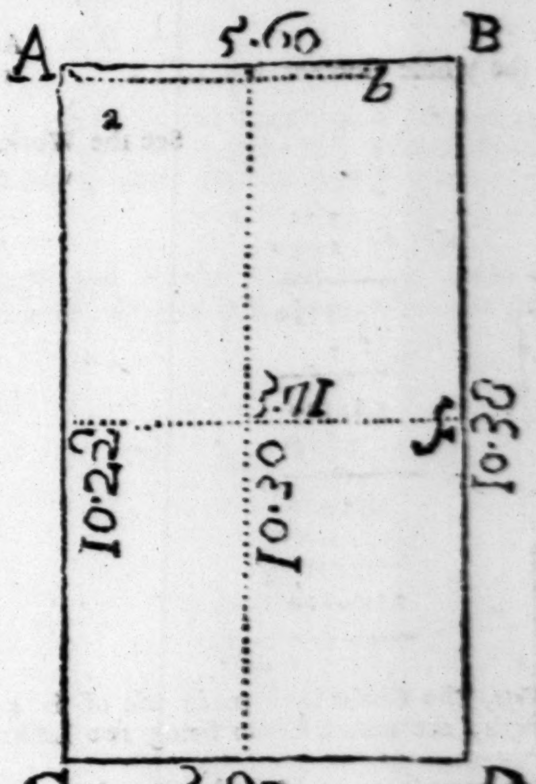
I Shall not here give the whole Art of *Surveying*, but such practical Rules only as may be useful to the Country Grasiers and Farmers, whereby they may find the true Content of any Piece of Land, and that by the Chain only; (and for Want of that, with a Pole, or Stick, of half a Rod in Length.

PRO-

PROBLEM I.

To find the Content of a Piece of Land in the Form of a right-angled Parallelogram, or Long-Square, or what is something near that Form.

TO know whether any Angle in a Field be a Right-angle, or not, you may take a piece of Board about 4 or 5 Inches broad, and an Inch thick, either round or square; and with a Saw cut two Kerfs crossing each other at Right-angles; and bore a Hole in the middle of the back Side, to put it upon the End of a Stick. This will represent the Instrument call'd a Cross.



Suppose you would observe the Angle A, to know whether it be a Right-angle, (or near thereunto) prick up your Stick, with the Cross upon it, a little Distance from the Fence, as at (a) and having set up two Marks, as at (b) and (c) of equal Distance from the Fence, turn one of the Slits directly towards (b); and then, if the other be directly pointing (c) it is a Right-angle.

To measure such a piece of Ground as this Figure above. If you measure round, and add the opposite Sides together,

gether, and take half the Sum, (if they be not equal) or else measure down about the Middle of the Length and Middle of the Breadth thus, the Side AB being measur'd, it will be 5.60; (that is 5 Chains and 60 Links) and the opposite Side CD. is 5 Chains 82 Links; the half Sum thereof is 5.71: And the Side BD is 10.38; and the Side AC 10.22; and the half Sum thereof is 10.30; (it will be the same Thing if you measure about the Middle of the Length and Middle of the Breadth) then multiply this mean Length and mean Breadth together, viz. 10.30 by 5,71, and the Product is 58.8130; which divide by 10. (because 10 square Chains is an Acre) by removing the separating Point one Place towards the Left-hand, and it will be 5.88130; that is, 5 Acres and .88130 Parts; which multiply by 4, and prick off 5 Places and it will be 3.52520; which 3 towards the Left-hand are 3 Roods; then multiply the decimal Parts by 40, and prick of 5 Places, and it will be 21.00800; which 21 towards the Left-hand are 21 Perches.

	A.	R.	P.
So the whole Content is ———	5	3	21

See the Work.

```
   5.71
  10.30
 ------
  17130
 571
 ------
5.88130
      4
 ------
3.52520
     40
 ------
21.00800
 ------
```

A. R. P.
5 : 3 : 21.

Note, The Chain here made use of, is 4 Poles, or Rods in Length; the whole Chain being 100 Links.

But because every Man that may have Occasion to measure a piece of Land can't procure a Chain, I will therefore shew how

how you may meaſure a Piece of Land only with a Stick of half a Rod in Length; that is, 8 Feet and 3 Inches: but in *Ireland* 'tis 10 Foot 6 Inches: Which Stick divide into five equal Parts, ſo will the whole Rod be divided into ten Parts, and will be thereby adapted to Decimal Arithmetick.

But becauſe each of thoſe Parts of the Stick are ſomething large, (each Part being 19 Inches and 8 Tenths, In *Great Britain* but 25.2 Inches in *Ireland*) it will be neceſſary to take your Dimenſions to half of one of thoſe Parts; and then, for that half Part, ſet 5 in the Place of Seconds, thus; ſuppoſe 3 Parts and a Half, ſet it down thus ,35.

Problem II.

LET us ſuppoſe a Field in the Form of a Long-Square, whoſe Length is 45 Rods 5 Parts and a Half, and the Breadth 31 Rods 4 Parts and a Half; What is the Content?

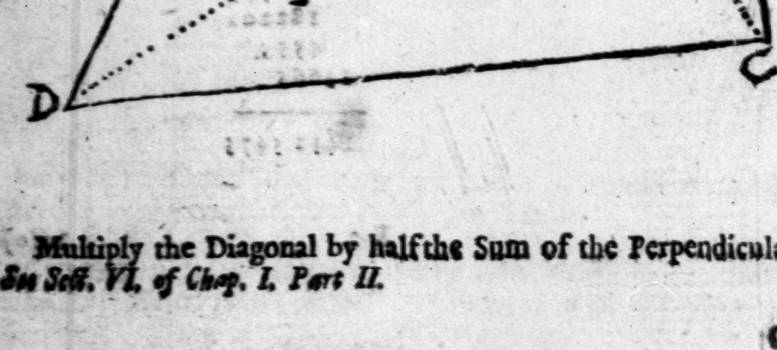
Multiply the Length and Breadth together, and divide the Product by 160, (becauſe 160 ſquare Rods is an Acre) and the Quotient is Acres.

```
      45.55
      31.45
    -------
      22775
     18220
     4555
   13665
  ---------
  1432.5475
```

16|0)143|2(8		A.	R.	P.
128	*Equit.*	8	3	32
4|0)15|2(3				
12				
32				

Problem III.

Suppose a Piece of Ground in the Form of a Trapezium; the Diagonal BD 13 Chains 60 Links, the Perpendicular CE 6 Chains 25 Links, and the Perpendicular AF 3 Chains 42 Links, what is the Content?



Multiply the Diagonal by half the Sum of the Perpendiculars *See Sect. VI. of Chap. I. Part II.*

CE

CE = 6.25
AF = 3.42

Sum 9.67

Half 4.83

```
13.60 = BD
 4.83
-------
  4080
 10880
 5440
-------
6.56880
      4
-------
2.27520
     40
-------
11.00800
```

A. R. P.
Facit 6 2 11

By Rods, thus ;

CE = 25 Rods.
AF = 13.68

Sum 38.68

Half 19.34

```
       19.34
        54.4 = BD
       ------
        7736
       7736
      9670
    ---------
16|0)105|2.096(6
       96
      ----
     4|0)9|2(2
          8
          --
          12
```

A. R. P.
Facit 6 2 12

To

To take the Dimensions of the Field.

Begin at the Angle B, and measure in a direct Line toward D; but when you come at E set up your Cross, and direct one of the Slits to D, and then look through the other Slit, and if it exactly hits the Angle C. then you are just in the Place where the Perpendicular will fall; but if it does not exactly hit the Point, move backwards or forwards till it does so; then measure the Perpendicular, and set down the Chains and Links, or the Rods and Parts; then continue your Measure towards D; but when you come to F, set up your Cross, and try (as above directed) whether you be in the Place where the Perpendicular will fall: Then measure the Perpendicular AF. and set down the Chains and Links, or Rods and Parts; then continue your Measure to D, and set down the Measure of the whole Diagonal, This Way of Measuring is very exact and true; but the common Way used by the Grasiers and Farmers. is to measure round the Field, and to take half the Sum of the opposite Sides for a mean Side; but the last mention'd Piece of Ground being measur'd so, will come to

R. P. R. P.
7 0 22, which is 2 10 more than the Truth.

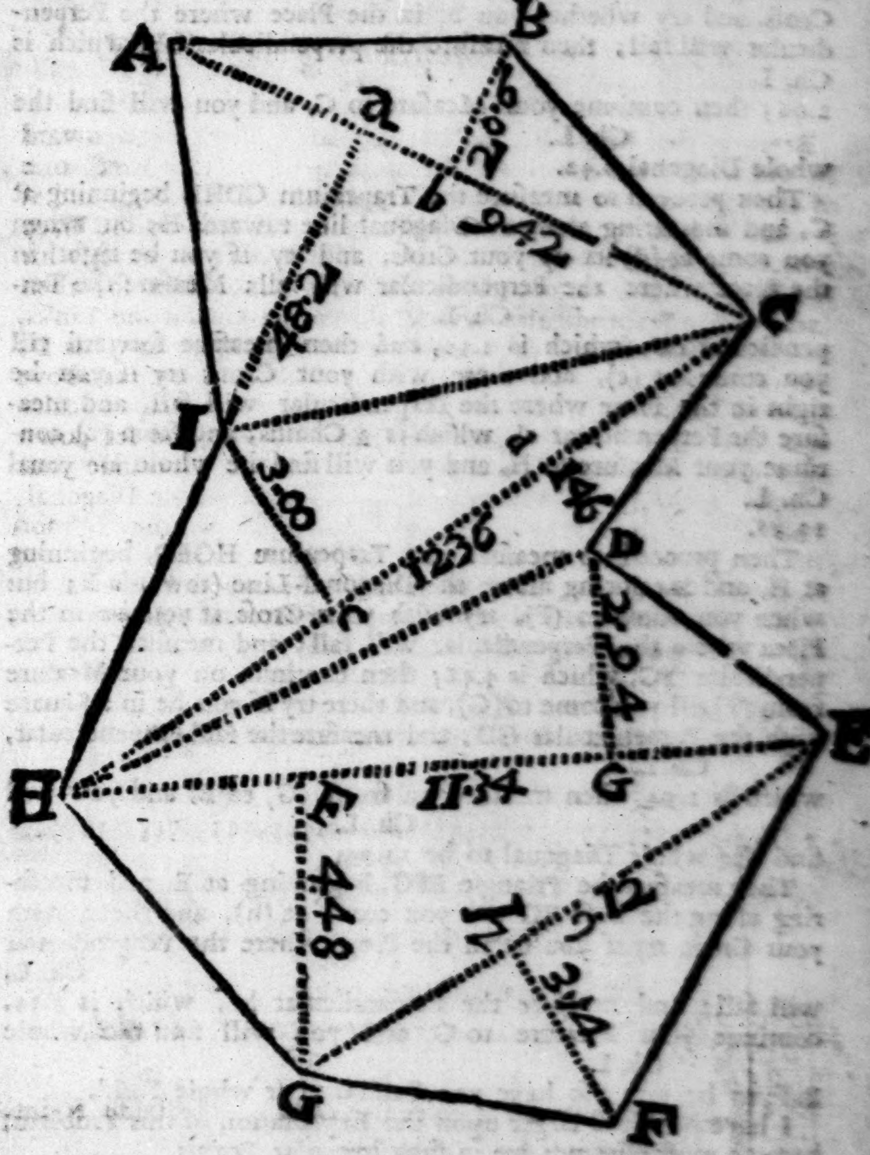
Problem IV.

How to measure an Irregular Field,

THE Way to measure irregular Land, is to divide it into Trapeziums and Triangles, thus:

First view over the Field, and set up Marks at every Angle, and by those Marks you may see where to have a Trapezium, as ABCI in the following Figure.

Then



Then begin and measure in a direct Line from A towards C; but when you come to (a) set up your Cross, and try whether you be in a Square to I, (as is before directed) and then
Ch. L.
measure the Perpendicular aI, which is 4.82, then measure forward

ward again towards C, but when you come to (b), ſet up your Croſs, and try whether you be in the Place where the Perpendicular will fall; then meaſure the perpendicular bB, which is
Ch. L.
2.06; then continue your Meaſure to C, and you will find the
Ch. L.
whole Diagonal 9.42.

Then proceed to meaſure the Trapezium CDHI, beginning at C, and meaſuring along the Diagonal line towards H; but when you come at (d) ſet up your Croſs, and try if you be right in the Place where the Perpendicular will fall: Meaſure the Per-
Ch. L.
pendicular dD, which is 1.46, and then meaſure forward till you come at (c), and there, with your Croſs, try if you be right in the Place where the Perpendicular will fall, and meaſure the Perpendicular cI, which is 3 Chains; and from (c) continue your Meaſure to H, and you will find the whole Diagonal
Ch. L.
12.36.

Then proceed to meaſure the Trapezium HGED, beginning at H, and meaſuring along the Diagonal-Line (towards E; but when you come to (F), try with your Croſs if you be in the Place where the Perpendicular will fall; and meaſure the Perpendicular FG, which is 4.48; then continue on your Meaſure from (F) till you come to (G), and there try if you be in a Square with the Perpendicular GD; and meaſure the ſaid Perpendicular,
Ch. L.
which is 2.94; then meaſure on from (G) to E, and you will
Ch. L.
find the whole Diagonal to be 11.34.

Then meaſure the Triangle EFG, beginning at E, and meaſuring along the Baſe EG till you come at (h), and there, with your Croſs, try if you be in the Place where the Perpendicular
Ch. L.
will fall; and meaſure the Perpendicular hF, which is 3.14, continue your Meaſure to G, and you will find the whole
Ch. L.
Baſe to be 9.12; ſo have you finiſh'd your whole Field.

I have been the larger upon the Explanation of this Problem, becauſe moſt Grounds lye in ſuch irregular Forms.

Caſt up the three Trapeziums ſeverally, and alſo the Triangle; and add all the ſeveral Areas together into one Sum, which will be the Area of the whole irregular Plot.

See

See the Work.

bB = 2,06	9.42	See Sect. VI. Chap. I. Part II.
aI = 4.82	3.44	
Sum 6.88	3768	
	3768	
half 3.44	2826	
	3.24048 = Area of ABCI.	

dD = 1.46	12.36
cI = 3.00	2.23
Sum 4.46	3708
	2472
half 2.23	2472
	2.75628 = Area of CIHD.

FG = 4.48	11.34
GD = 2.94	3.71
Sum 7.42	1134
	7938
half 3.71	3402
	4.20714 = Area of HGED.

Base = 9.12	
Half = 4.56	See Sect. V. Chap. I. Part II.
Perpend. 3.14	
1824	
456	
1368	
1.43184 = Area of the Triangle EFG.	

1.43184

1.43184 = Area of the Triangle EFG.
3.24048 = Area of ABCI.
2.75628 = Area of CIHD.
4.20714 = Area of HGED.

Sum 11.63574 = Area of the whole.
4

2.54296
40

21.71840

	A,	R.	P.
Facit	11	2	21

F I N I S.

READER,

Seeing this Book was corrected at distant times and by different Hands 'tis no marvel if therein appears a diversity of Spelling without prejudice to the Sense, viz. *When one thing is Spell'd both* Frustum and Frustrum, *another is called* both plain and Plane, Also

Page	Line	For	Read
119	20	The	TS×
124	9,10,12	×	+
129	12	$\frac{11}{12}$	$\frac{1}{12}$
145	3,7	19846	19.846
189	3	$\overline{3}$	$\frac{1}{3}$
289	11	38565	3.8565
307	6,7	169617	16.9614
		487	48.7

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